MODELS

Andrew Kalotay, president of Andrew Kalotay Associates, explains what single factor interest rate models can — and can't — do for you

The Problem with Black, Scholes et al.

It's all the fashion today. Bond traders, dealers and end-users are valuing a tremendous variety of bonds containing embedded options using one-factor interest rate models. Indeed, these days they are used to value everything from plain vanilla callable bonds to convertible bonds, structured notes and even CMOs[1]

Unfortunately, this rampant use of one-factor models adds up to widespread misuse. One-factor interest rate models are only reliable for pricing instruments that have simple structures and whose value depends exclusively on interest rate movements; they have serious limitations when they are used to price even moderately complex instruments. This article will explain how single factor interest rate models work and identify when they should — and should not — be used.

Valuation 101 and Black Scholes

One-factor interest rate models are just a subset of the many one-factor models that are commonly used to value a whole raft of financial instruments. But the basic approach is always the same. Assume that a particular market variable follows some random behavior over time and use the principles of arbitrage-free pricing (AFP) to value a related security. Often the price of the security underlying the instrument being valued is used as the "single factor."

One example is the model developed by Black and Scholes to value call and put options on dividend-free stocks. This model assumes that the underlying stock price will follow a lognormal random walk as time passes.

In the case of options on fixed income securities, however, using the value of the underlying security as the "one factor" leads to serious problems. This is particularly problematic with callable bonds, where the issuer has the right to purchase the bond after a certain number of years of call-protection at a contractually specified strike price. In general, issuers exercise such options during particularly low interest rate markets and reissue lower coupon values.

Options on bonds cannot be priced by directly modeling the price of the underlying bond because bond prices don't follow random walks. Although a bond's price will vary in some random fashion as time elapses it will always return to par as the bond's maturity approaches. This is clearly not a feature of the random walk. Early attempts to value options on bonds by directly modeling the bond's price were doomed to failure.

If the underlying bond's price cannot help price the humble callable bond, then what can? Well, the most successful — and popular — approach involves the modeling of something even more fundamental than the underlying bond's price, namely, the behavior of short-term interest rates.

The Ho and Lee model was the first attempt to use short-term interest rates to gauge the price of callable bonds, but it wasn't very realistic. The model assumed that short-term interest rates followed a random walk — an approach that allows negative interest rates.

Several solutions to this deficiency have been proposed. In the most popular models, the short-term rate follows a lognormal random walk — a type of walk that never goes below zero. With a good model of interest rate behavior in place, the principles of arbitrage-free pricing can be used to value many fixed income securities just as Black and Scholes used a stock's random behavior to calculate stock option values.

The building blocks

Most of today's one-factor interest rate models use three different parameters to describe the random behavior of interest rates. These parameters are drift, volatility and mean reversion. Each parameter affects in a different way the sorts of interest rate scenarios likely to arise from the random process.

(1) Drift

Drift is any bias that may exist in the random movement of short-term interest rates. For example, if today's yield curve were positively sloped, reflecting a market expectation that interest rates are likely to rise in the future, there would be a bias in the random process of favoring rising rates. This drift variable, of course, is time-dependent, and most "drift estimates" are typically based on today's yield curve or term structure.

(2) Volatility.

Volatility, in simple terms, is the variable that determines the magnitude of random changes in short-term interest rates. The larger the volatility, the greater the randomness of short-term interest rates In other words, the greater the volatility, the greater the range in which interest rates are likely to fluctuate In general, the larger the volatility component, the greater the value of the option on a bond.

(3) Mean reversion

Mean reversion refers to how likely it is for the short-term interest rate to be pulled back, over time, toward its mean value. (The mean value at some future time is roughly the sum of the initial short-term rate and the estimated drift). Mean reversion — which can be likened to the force of a stretched rubber band — increases in strength along with the magnitude of interest rates' deviation from the mean. Thus, the more "oddball" a particular interest rate is, the greater its associated mean reversion. Mean reversion affects the "term structure" of volatility. As mean reversion levels increase, the volatility of longer maturity yields decrease.

All models in widespread use incorporate time-dependent drift to explain the current shape of the yield curve. The data needed to run these models is easy to get since Treasury yields are available over a wide range of maturities Some models also allow for the time-dependent volatility and mean reversion. They require much more data to use. Indeed, these models usually require data that is not readily available to many users — the prices of a wide assortment of fixed income options, for example Moreover, this sort of data is not as "clean" as one would like because the requisite securities are often thinly traded and have wide bid/ask spreads.

What does all this tell us? Do not apply single factor models to securities that require multi-factor models Convertible bonds, for example, should be valued according to a model that considers both the behavior of interest rates and the issuer's stock price. Similarly, a multi-factor model would be required to accurately value securities like spread options whose payout depends on the shape of the yield curve.

One-factor models have limitations even within purely interest rate-dependent securities This stems from the fact that (for a given random process) the entire yield curve can be calculated at a future point in time once only the short-term rate is known — this is just the nature of a single factor model. The shape of the yield curve is therefore determined by the level of rates. It doesn't make sense to use a single factor model to value any security whose cashflow is linked to the shape of the yield curve. Examples of such securities include spread options and adjustable-rate preferred shares.

To conclude, while one-factor interest rate models are certainly useful and pervasive, their accuracy — and appropriateness — must be carefully and regularly evaluated. ■