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OPTIMUM BOND CALLING AND REFUNDING

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ABSTRACT. Long-term corporate bonds have traditionally been issued with a *call option*, whereby the issuing company reserves the right to *call* the bond, that is, to redeem or buy back the bond prior to maturity. Usually the purpose of calling the bond is to be able to *refund* it, that is, replace it with one having a lower interest rate, or *coupon*. During 1976-77, when interest rates had dropped significantly from the rates of 1969-70 and 1974-75, Bell System companies called or refunded over $2 billion of long-term debt. This paper describes how analytical techniques developed at Bell Labs and AT&T played a significant role in these refundings.

Introduction

In recent years the bond market has attracted increasing attention from analysts and investors, particularly institutional money managers. In response, corporate financiers and their advisors have developed improved tools for managing and controlling their portfolios of debt, whether they be net borrowers or lenders.

Big numbers are involved. Outstanding bonds in the United States total $1.5 trillion, with U.S. Treasury issues accounting for $600 billion, other Federal agencies $200 billion, state and local issues another $300 billion, and corporate bonds over $400 billion. About $150 billion of the latter represents the debt of public utility companies, with the Bell System itself (American Telephone and Telegraph Co. (AT&T), its subsidiaries and associated companies) accounting for $35 billion. Over $10 billion of new corporate debt comes to market each year, and the Bell System's share is over $2 billion. Predominating among the lenders are institutions with preferential income tax treatment, such as pension funds (which are tax-exempt) and life insurance companies (which have relatively low income tax rates).

We are primarily concerned with *long-term* bonds, those with lifetimes or *maturities* of 20 years or more (40 years is the most prevalent Bell System maturity). The bond's *par value* or *face amount* is the principal amount payable on maturity. The annual interest payment, expressed as a percentage of face amount, is often referred to as the *coupon*.

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Although long-term interest rates are not as volatile as common stock yields, they still exhibit considerable variability. Figure 1 shows the history of interest rates on new long-term prime (Aaa) public utility bonds from 1969 up to the present. Two pertinent conclusions that could be drawn from this figure are that the pattern seems quite random, and that those companies that sold bonds at the higher rates might wish to retract that action at some future time.

FIGURE 1. Rates on New Long-Term Aaa Public Utility Bonds Since 1969.

This brings us to the subject of our investigation, a very frequent feature of long-term corporate bonds known as the *call option*, which gives the issuing company the right to prepay the debt, or buy the bond back before maturity. When this happens the company is said to *redeem* or *call* the bond. The most common use of this option has been to *refund* the debt when interest rates have fallen by issuing new bonds at a lower rate and using the proceeds to pay off a higher-coupon callable bond. Since about 1960 this call option has usually been limited by a call *deferment* provision which prohibits the option being exercised too soon after issuance (five years for Bell System issues). This call deferment is one form of call *protection* for the lender. An additional measure of call protection is furnished by the fact that the preset purchase price includes an extra few percent above face amount called the *call premium*, which also partially compensates the investor for the reduced interest income that usually results from a call.

The main objective of our study, which began in late 1975, was to determine how existing call options should be used — that is, when should a bond be called? A related basic question was whether the inclusion of a call option in new bonds was beneficial to the company.

For the purpose of this paper, we will focus on two Bell System bond issues for which the call decision was particularly influenced by our analyses. These were the New Jersey Bell Telephone Company 9.35% bonds and the American Telephone and Telegraph Company 8.70% bonds, both of which were issued in 1970 and called in 1976. These, along with other calls and refundings of long term Bell System bonds during 1976-77, are indicated in Figure 1 and keyed to the Summary Table (Table 1).

TABLE 1. Summary Table.

	Coupon	orig. amount (millions)	called or tendered	yield advantage	after-tax percent saved	efficiency
1. Southern	9.05	150	150	76	3.7	69
2. New Jersey	9.35	100	100	135	10.3	99
3. New York	9.25	150	150	80	5.2	77
4. Mountain	9.00	150	150	111	8.3	95
5. AT&T*	8.70	350	350	~110	~7.0	~97
6. Tenders:						
Indiana	10.00	80	55.7	185	11.1	83
Northwestern	10.00	150	120.9	180	10.7	82
South Central	10.00	225	193.9	175	10.1	80
New England*	9.50	175	120.0	~118	~6.0	~75
7. Pacific	9.125	150	150	80	5.7	89
8. Mountain (tender)	9.75	175	114.9	177	11.3	89
9. Indiana	9.00	80	80	77	5.4	79
10. AT&T*	8.75	1569.3	854.7	~70	~3.0	~69

*No refunding issue; yield advantages are estimates.

A case history — New Jersey Bell's 9.35's

In June 1970, New Jersey Bell, one of the operating telephone subsidiaries of AT&T, sold $100 million of 40-year bonds with a coupon of 9.35% payable semiannually. The bonds were priced at face amount, and after commissions New Jersey Bell received $991.25 per $1000 bond. As is customary, we will express prices as percentages of face amount, e.g., the net proceeds were 99.125. Any time after June 1, 1975 (the call deferment) the bonds could be called at a declining schedule of prices, initially 108.01 for the year beginning June 1, 1975, plus accrued interest. Each June 1 the call price decreases until it becomes 100 on June 1, 2005, five years before maturity. (In 1970 Bell System bonds were standardized so that for an N-year bond, the call premium at five years would be $(N-10)/(N-5)$ times the algebraic sum of the coupon and premium or discount (with a discount taken as negative), and would decline by $1/(N-5)$ of this sum per year, becoming zero five years before maturity.)

In June, 1975, when the bonds first became callable, a similar Bell System company sold bonds which yielded only 8.65%. If New Jersey Bell had called the 9.35's and issued an equal amount of 8.65's also maturing in 2010, then the interest savings before taxes would have been $700,000 per year for 35 years. The main expense would be the call premium of $8,010,000. (We neglect for now the other costs of refunding, which are relatively small.) The gross undiscounted savings in interest due to the call is $700,000×35=$24,500,000. The call premium of $8,010,000 offsets the savings and leaves a gross savings of $16,490,000. But this figure is not very useful for financial planning and analysis since it ignores both taxes and the time value of money.

Since interest payments are deductible when calculating income taxes, at the marginal corporate tax rate of about 50% the true cost of interest is only about half the interest payments. Thus the net savings due to reducing interest by $700,000 a year is only $350,000 a year. What some analysts have apparently overlooked is that the call premium, although representing a capital gain to the bondholder, is immediately deductible by the issuer of the bond. Thus the net expense of the call is also only about half the amount of the call premium. The total net savings at a 50% rate is then only half of $16,490,000, or $8,245,000.

To account for the time value of money we wish to apply an appropriate discount rate to the future net cash flows due to the refunding, i.e., the $350,000 per year savings for 35 years. This figure is the difference between the future net flow which was cancelled by the call, namely $4,675,000 for 35 years plus a $100 million payment in 2010, and the future net flow required by the new refunding issue, namely $4,325,000 per year for 35 years plus a $100 million payment in 2010. We assume that the refunding issue is fairly priced to the company, so that it has present value $100 million. This implies that the appropriate discount rate for flows in this risk class in June, 1975, was 4.325%, the after-tax cost of debt to the company at that time, as evidenced by the cost of new debt. If we concede that the old and new flows have essentially the same risk, the same discount rate is appropriate for each and for their difference. (At 4.325% the new bond has a net value of zero, so that an equivalent method is to use the new bond only to determine the proper discount rate, which is then applied to the net flows of the old bond which are being canceled.)

At a 4.325% discount rate the future savings are worth over $6 million, which exceeds the net call premium of about $4 million handily. Therefore a refund at 8.65% would be profitable, with an after-tax savings of about $2 million, or about 2% of face amount. The zero-savings or ''breakeven'' discount rate is 4.45%, corresponding to a pretax rate of 8.90%.

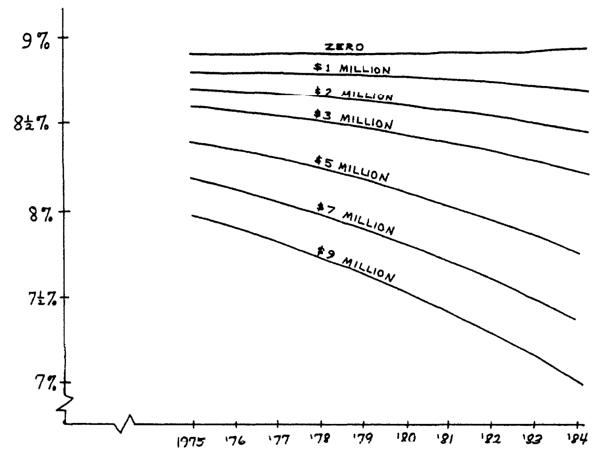
But New Jersey Bell did not take advantage of this opportunity, and interest rates did not stay low enough to be tempting very long. In the latter part of 1975 rates went back over the 9% level.

New Jersey Bell was scheduled to sell new debt in February of 1976, and in the early weeks of 1976 rates dropped sharply toward 8.50%, a level at which refunding of the 9.35's would be profitable. Just six days before the scheduled sale, New Jersey Bell announced that it planned to increase the size of the February issue and call the 9.35's. But New Jersey Bell officials were still ready to listen to new ideas on the subject.

By that time we had extended the profitability analysis by computing profitabilities at many future times, discounting back to the present, and drawing level profit lines by connecting points of the same discounted profitability. In this way one could judge the relationship between future profitability and forecasts, mathematical or intuitive, of the future course of interest rates.

Figure 2 shows the curves that were calculated for the New Jersey Bell 9.35's. Along each one of these contours the discounted value of the refunding is constant. This indicated that there was no particular advantage to hurrying and, indeed, higher profits could very likely be obtained by waiting.

FIGURE 2. Constant-Savings Contours for New Jersey 9.35's.

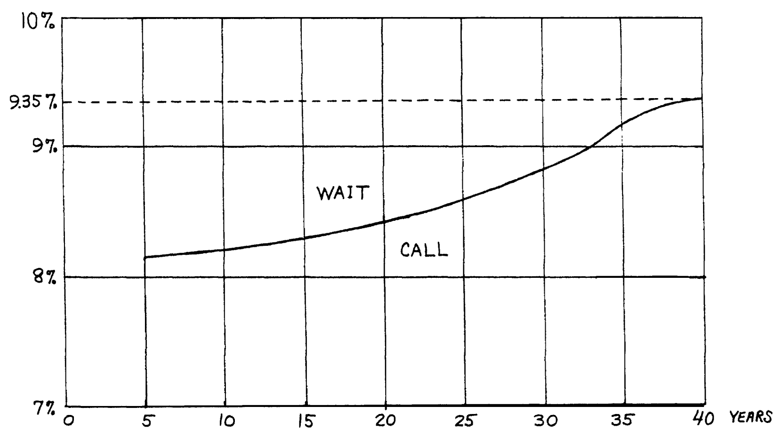


Based in large measure on this information, New Jersey Bell reconsidered and decided to raise only the necessary new money and pass up the refunding at that time even though it would have been profitable. Since the new bonds yielded only 8.34%, what had been declined turned out to have been a 1.01% refunding advantage (more commonly, a 101 "basis point" refunding, with 100 basis points equal to 1%).

Soon after New Jersey Bell's new bonds were placed we had substantially completed a bond-refunding decision model which was based on a geometric random walk model of interest rates and stochastic dynamic programming. Via the dynamic programming it was possible to consider not only the profitability of a single refunding opportunity, but to compare it with all possible future refunding opportunities. The primary output of this model was a single curve (see Figure 3), dividing the region where an immediate refunding was optimal from that where profitability was expected to be better in the future. For the New Jersey Bell 9.35's, at five years the dividing line was at about 8.10%, so the decision to pass up a refunding at 8.34% was "correct" in terms of the assumption of the model.

We discussed with New Jersey Bell officials how the random walk theory and profitability formulas interacted to give the curve, and how our figures supported their decision to wait. These results were subsequently presented as part of a discussion of New Jersey Bell's experience at the Bell System Assistant Treasurers Conference held in May of 1976.

FIGURE 3. Calling Curve for New Jersey 9.35's.



Although rates were too high during the summer for refundings to be attractive, AT&T used the time to complete development of a workshop on refunding fundamentals for Bell System treasury officials. For this we were asked to prepare a presentation on the timing decision for refunding. The workshop subsequently was given on three occasions around the country. Each subsidiary represented at this workshop was given a plot of the "stopping" curves for its outstanding bonds.

When rates moved lower in the early fall, New Jersey Bell decided to proceed with a September refunding of the 9.35's. When the bid came in, it was not for the 8.10% that was expected but only 8.00%, so that New Jersey Bell had reduced its interest charges by 1.35%, and improved on the refunding passed up in February by 34 basis points. This result considerably enhanced the credibility of our model and optimization procedures.

Discussion

The New Jersey Bell experience shows that the call option is a very valuable right. But from the mid-1960's through the early 1970's its value was often overlooked, probably because the uptrend in interest rates provided no refunding opportunities. Likewise, the cost of the call option was not generally acknowledged. (By the cost of the option we mean not the cost of exercising the option, which is basically the call premium, but the cost of having it included with the bond to begin with.) The evidence indicates that this cost shows up as a higher coupon required for a callable bond than for a noncallable bond. Comparing noncallable Treasuries with callable prime corporate bonds reveals an interest rate differential of 25 basis points up to 100 basis points and more, but this is usually attributed to the higher default risk of the corporates.

Our techniques allow us to study and evaluate these call values and costs. We find that a tax-exempt investor, in order to compensate himself for the risk of calling, should demand a coupon approximately 0.30%, or 30 basis points, higher than he would require on noncallable bonds. Thus on its $35 billion of debt, mostly long-term callable bonds, the Bell System is paying approximately $100 million before taxes, and $50 million after taxes, each year because its bonds were issued with a call option.

What is obtained from this 30-basis-point "rent" is an option with a value at issuance of the bond which we estimate to be between 3% and 4% of face amount. The present value of the extra 30 basis points, after taxes, is about 2%, leaving a net value of the call option at issuance of between 1% and 2%. For instance, on the $1790 million of Bell System long-term callable debt issued in 1975, the present value of the higher interest after taxes was approximately $38 million, while the total option value was about $63 million, leaving a net value of the call option at issuance of approximately $25 million.

We estimate that at the end of 1975, when we began our analysis in earnest, the total value of the call options on the existing $30 billion of Bell System debt was in excess of $300 million, or 1% of face amount, at the then-prevalent rate of 9%. Most of this $300 million was attributable to the bonds with coupons of 7% and more issued in the previous six years, with a heavy concentration in those bonds paying 8.70% and more issued during 1970 and 1974. The drop in rates to the 8.5% level early in 1976 pushed the total option values up to $500 million, and later in the year, when rates reached the 8% level, the value exceeded $800 million.

The state of the art — circa 1975

Financial tradition provides relatively little guidance on how best to use the call option, that is, when to call. Of course, those who have confidence in forecasts of interest rates will want to wait until rates have "bottomed out." But even when forecasts have been supported by the majority of expert opinion, they have been wrong often enough to cast doubt on their reliability.

A contrasting financial tradition is the "trigger point" theory, which holds that a refunding should be accomplished when the coupon on the new issue reaches a certain "trigger point" compared to the coupon on the issue to be refunded. Most quoted of these folk prescriptions is the "100 point" rule, which asks for a new coupon 100 basis points, or 1%, below the refunded issue's coupon. An alternate formulation is a similar but less quoted "75 point" rule. It is hard to track down where these came from, but whatever their origins and justifications, when concern about refundings arose in 1976, the 100-point rule was most often quoted as being the accepted wisdom.

There has also been considerable academic work on the subject of bond refunding. Hess and Winn's book [4] contained much valuable material but had little long-range impact. More often cited was Bowlin's fundamental paper [2], in which he not only surveyed recent refunding experience but also strongly endorsed the use of the after-tax cost of debt as the proper discount rate for after-tax savings. Several authors, notably Pye [5], have pointed to the usefulness of dynamic programming for analyzing refundings, but the examples that they discuss are not very realistic. For a critical review of other previous work, we refer the reader to our forthcoming article [3].

Random walk assumptions

The foundation of our analysis is the random walk hypothesis. This holds that changes in interest rates are difficult to predict, and future rates will vary about the current values in ways that cannot be determined from the past. We do not exclude the possibility that knowledge other than that of past rates may aid in forecasting, but we hold that such information is usually quickly incorporated into the current rate so that its marginal value is negligible.

Empirical studies indicate that a geometric random walk is a good model of interest rates (that is, the difference of logarithms of successive rates appear to be independent, identically distributed random variables), with the monthly σ (standard deviation) being about 0.015, and the yearly σ about 0.05. If the interest rate at the beginning of the year is 8.00%, then, 0.015 of 8.00% is 0.12%, and 0.05 corresponds to 0.40%, so we expect that after one month there is a 68% chance (plus or minus one σ) of rates being between 7.88% and 8.12%, and after one year a 68% chance of rates being between 7.60% and 8.40% (approximately).

We represent the random walk as the accumulation of tosses of a fair coin, or Bernoulli trials. Over each interval of length Δt the logarithm of interest rates is increased or decreased by an amount Δx with probability one-half each. Values are computed by dynamic programming, beginning at the maturity of the bond and working backwards to the time of issuance. The value for a particular time and interest rate is the average of the values at the two "successor" points Δt later, discounted back over Δt at the current interest rate, unless "calling" is a better value, in which case the "calling" value is substituted.

For all our results presented here, we use a Δt of 1/12 (one month) and a Δx^2 of $(0.05)^2/12$, corresponding to a yearly σ of 0.05. For a 40-year bond this provides 480 decision intervals and over 100,000 evaluation and optimization points. The algorithm is efficient enough that numerous cases can be run at modest cost.

Technical innovations

Several innovations contribute significantly to the practicality of our results. A primary one concerns which rate is taken as the "standard" rate, that is, the one undergoing the random walk. Operationally, it is the rate used to discount values between decision points, but it turns out to be extremely close to the rate demanded on a *long-term, noncallable* bond. For if we use our computational model to determine the appropriate coupon for a 40-year noncallable bond, we find that when the standard rate at issuance is 8% to 9%, the break-even coupon is only about 8 to 10 basis points below the standard rate. This small difference is due to the variability of the standard rate over the 40 years.

Similarly, the spread between this standard interest rate and the coupon required on a long-term *callable* bond may be computed from the model. With the variation in interest rates that we assume, we find that for a long-term Bell System callable bond, tax-exempt investors will demand a coupon between 20 and 25 basis points above the standard interest rate. Together with a noncallable bond having a coupon 5 to 10 basis points *below* the standard rate, this yields the difference of about 30 basis points between callable and noncallable coupons that we have quoted earlier. Such a spread is consistent with observed Bell System-Treasury yield differences.

A second innovation of our analysis is a considerable de-emphasis on the details of the refunding issue. If the new bonds are priced fairly, then the present value of the obligations incurred will be closely matched to the sale price. As indicated in the discussion of the New Jersey Bell case, the refunding issue may be considered as simply determining the proper rate of discount to be applied.

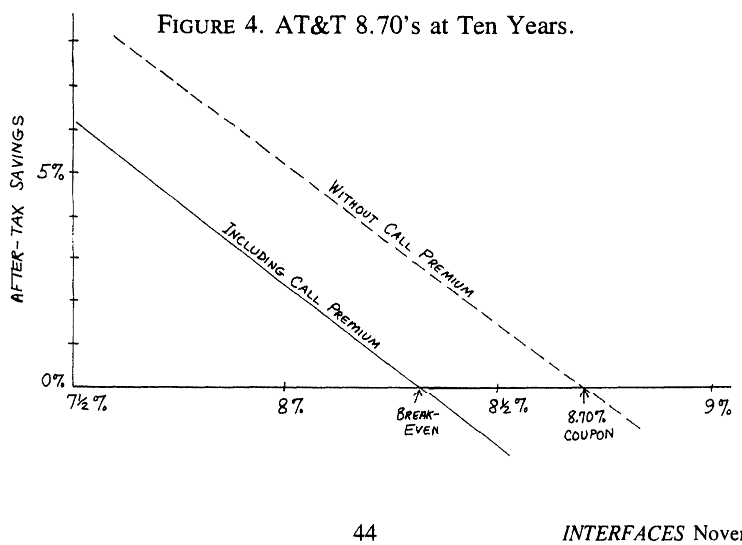
A third innovation is our treatment of taxes. Not only do we use all flows in their after-tax form, but we also adjust discount rates to a net after-tax basis. The borrower, a taxable corporation, has its discount rates scaled by its marginal tax rate and typical commissions. Lenders are assumed to be tax-exempt and commission-free.

An example — AT&T 8.70's

To illustrate our procedures we reproduce here material that we presented to key AT&T Treasury officials on December 9, 1976. The example, carefully chosen for that occasion, was the AT&T 8.70's of 2002, a 32-year bond issued in 1970 in the amount of $350 million. Since AT&T bonds yield about 25 basis points less than subsidiary bonds, we believed that the 8.70's should be considered for calling along with subsidiary issues yielding about 9%. Prior to the briefing, AT&T officials had been uncertain whether the 8.70's should be called. But within the next six days, the calling of the 8.70's had been thoroughly studied, recommended, approved by the AT&T Board of Directors, and publicly announced.

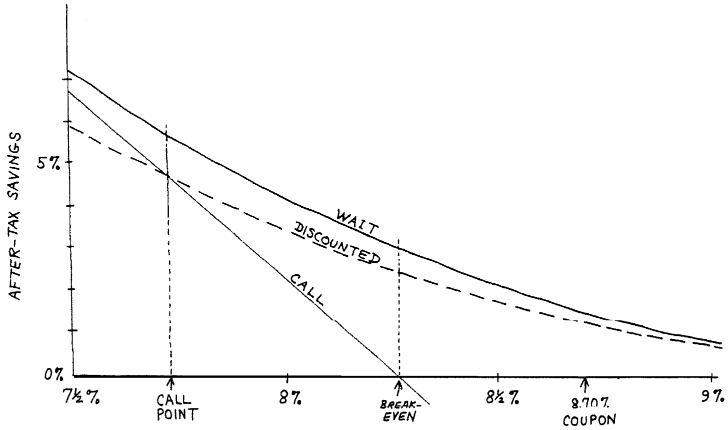
For simplicity we permit only two calling opportunities, at 5 years and at 10 years, instead of anytime after five years which is what the indenture actually allowed. Since year 10 is the last calling opportunity, the decision at that time is made myopically. Thus we consider year 10 first. For each pretax interest rate one can compute as of year 10 the present value of the interest savings obtained by refunding. This is shown as the dashed line in Figure 4. Reducing this by the after-tax cost of the call premium gives the solid line in Figure 4. The optimum strategy for this final decision time is to call the bonds if rates are below the "break-even" of 8.32%, and to forego the call if interest rates are too high for profitability. The value at year 10 for this optimum strategy is given by the savings curve to the left of the break-even point, and by the "zero" line to the right.

FIGURE 4. AT&T 8.70's at Ten Years.



At year 5 the analysis is more complex, since one may wish to pass up a profitable refunding at year 5 in hopes of obtaining a more profitable one at year 10. At each level of interest rates at year 5, given a stochastic model of interest rates, there is an expected savings to be obtained by waiting to year 10. This is indicated by the highest, solid curve in Figure 5. Note that the expected savings are positive even for interest rates higher than 8.70%, since the savings are never negative and there is a positive probability that rates will drop enough in five years to allow a profitable call.

FIGURE 5. AT&T 8.70's at Five Years.

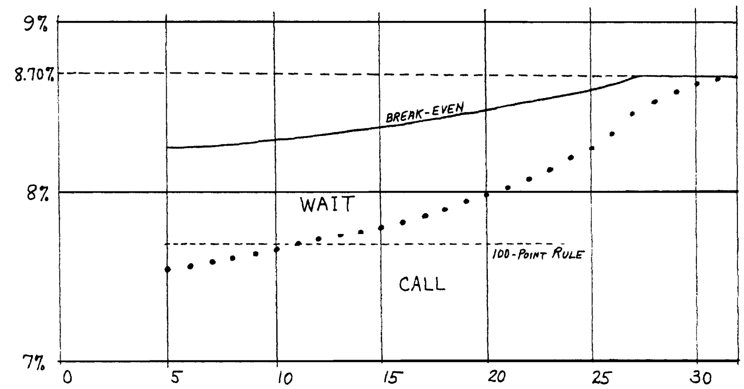


To obtain the value in year 5 of these possible savings in year 10 one must compute a present value by using the after-tax stochastic discount rate beginning at the year-5 interest rate. This is the lower, dashed curved line in Figure 5, and it represents the value of waiting. The diagonal solid line in Figure 5 is the savings from calling in year 5, computed as was the analogous quantity in year 10. The optimum refunding decision in year 5 is then to call unless the value of waiting exceeds the savings obtained by an immediate call. In Figure 5 the "call point" of 7.72% marks where the waiting and calling curves cross.

The general procedure is just the foregoing using hundreds of decision intervals instead of just two. At each decision time, the value of an immediate call is compared with the value of waiting at least one more time step. The discounting over the intervening step uses the after-tax standard rate existing at the earlier step. As the interval between decisions decreases, the calling and waiting curves become tangent at the call point.

Figure 6 shows the graph of the calling points (only one per year) that one obtains for the AT&T 8.70's, after all the adjustments and modifications have been included. We see that the points are about 115 basis points below the coupon when the bond first becomes callable. Figure 5 differs from Figure 3 because the coupon, call premiums, and maturity are different, and in Figure 3 the annual call points have been connected by a continuous curve.

FIGURE 6. Call Points for AT&T 8.70's.



Efficiency

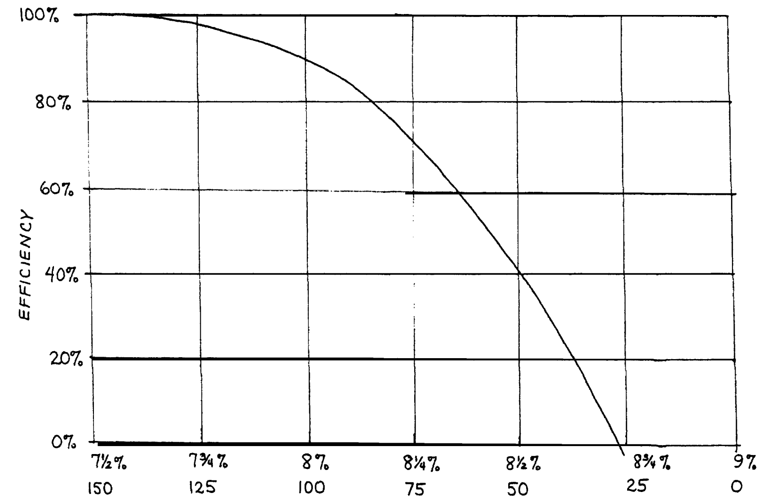
Between the call point and break-even in Figure 5 the immediately available savings are positive but less than optimal. To aid in the evaluation of calls in this region, we developed the concept of the *efficiency* of a call or refunding. There was a need to move beyond the simple black-and-white world of "call when you first hit the curve" which was the heritage of optimal stopping theory. An initial effort was to compare the savings available at a suboptimal point with the savings that would be achieved if rates were at some optimum point, such as the call point for that same time. But this was clearly an unfair comparison since there was no assurance that a particular point on the calling curve would ever be reached.

A better scheme would be to compare the savings immediately available with the savings expected if one used the optimum strategy starting at that time and standard interest rate. This expected savings was clearly the value of the call option for that time and rate.

We still had the choice of expressing the deviation from optimality either as a difference or a ratio. From the standpoint of Bell System practice there were strong arguments for using a ratio, expressed as a percentage from 0 to 100. This would make the measure an *index*, and various indices scaled from 0 to 100 play an important role in Bell System operations.

Figure 7 shows the efficiency curve for a 40-year 9% bond at five years. This is typical in that the tangency of the call and wait curves at the call point translates into the tangency of the efficiency to 100% at the call point. Because of the very high efficiency of the 100-plus calls, we have come to realize that the exact location of the call point is really not that important. Anything significantly over 100 basis points is pretty good. The 100-basis-point rule is about 90% efficient.

FIGURE 7. Efficiency for a 40-year 9% Bond at Five Years.



In late 1976 the efficiency concept proved particularly valuable in evaluating a proposed *tender offer* for a group of high-coupon bonds which were less than five years old and not yet callable. For this situation there was no conventional guideline, such as the 100-point rule for calling. A new tool was needed, and efficiency filled the bill. By calculating efficiencies, we could relate the possible savings on a tender offer to what could be expected from future call opportunities. We also compared the efficiencies for a range of possible tenders with efficiencies achieved on earlier refundings, and we were able to report that on that basis the proposed tenders were superior to some of the refundings previously accepted. Based on this information, as well as numerous other factors, tender offers for the issues were made in January of 1977.

Implications of the option value and cost

We have previously observed that the coupon required on a callable bond of Bell System type is a little less than 25 basis points above our standard rate, the "long-term noncallable" rate used for discounting. For simplicity we have standardized this spread at 25 basis points for the work reported here.

Therefore when the corporate bond "market rate" is the main indicator available of the level of interest rates, these 25 basis points must be subtracted in order to obtain the proper rate for discounting. Thus our New Jersey Bell calculations were deliberately inaccurate, although they were consistent with the state of the art prior to our work. If the going rate on a *callable* Bell System bond was 8.65% in June of 1975, then the current standard rate was not that figure, but about 25 basis points lower, or 8.40%. An 8.65% replacement bond, since it would be callable, would not in general produce the future stream of payments quoted, since the stream would be considerably altered if the 8.65% bond was subsequently called. If one wanted a guaranteed stream for comparision purposes, one needs a *noncallable* bond of like quality, which our analysis indicates would sell for closer to 8.40% than 8.65%.

Although our calculations all use the standard rate, we plot the equivalent callable bond ''market rate'' obtained by adding in the 25 basis points. Thus when we say that the stopping point for the New Jersey Bell 9.35's was at 8.10% after six years, the 8.10% refers to a callable replacement bond. The actual standard rate at which the call should be made was 7.85%. This 25 basis points makes our figures for the savings noticeably higher than those computed using the market rate as if it were the rate required on a noncallable bond.

One of our original concepts was that the replacement bond could be largely ignored as long as it was fairly priced. But as we have shown in a related paper [3], when a bond is *callable*, it can be fairly priced to the lender while still having a positive net value to the borrower, and *vice versa*, if the borrower has a higher marginal tax rate than the lender. For the cases that we are considering — corporate borrowers and tax-exempt lenders — a callable replacement bond will not in general have a zero net value.

Since the replacement bond is usually callable, we have included an allowance for its net value in computing the calling points and curves (it raises the curves about 10 basis points), but we have not included it in calculating refunding savings or efficiencies.

One way of expressing the net value of the callable bond is to note that the 25 basis points above the standard rate is what the lenders require for the call option. Corporate borrowers should actually be willing to pay as much as 50 basis points over the standard rate. In our work, however, we assume that the bond is sold at the lenders' break-even so that the borrower gets the whole benefit. This is particularly appropriate for regulated borrowers.

Conclusion

We have described several cases in which our analyses contributed to the saving of millions of dollars for the Bell System, and we expect that our work will also prove significant in the future. Much of its influence will be through the excellent precedents that we have helped to establish of how profitable a refunding can be.

Often we argued that refunding should be delayed in the expectation of larger savings in the future. An opposing view was that if a 100-basis-point call was the ''ideal'' (which we disputed), then there was little harm in shading it to 90, 80, or 70 basis points. We pressed the viewpoint that in an 8% to 10% market, the 100-basis-point rule was a minimum, not a maximum.

The Bell System calls have spanned almost the whole spectrum of source of funds and justification: standard calls (75-100), optimum calls from both refunding issues and cash-on-hand, and calls paid for from equity for restructuring of capital. We hope that from this experience the finanical community has learned much about techniques for using the call option, especially the usefulness of random walk theory and stochastic dynamic programming via computer for the calculation of optimal call points and efficiencies. We also hope that we have succeeded in putting refunding practice on a sound theoretical basis, and helped to reconcile and unify the views of financial theory and practice relative to refunding.

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TIMS MEETING CALENDAR ORSA

Dates	Location/Hotel	General Chairman
*March 13-15, 1980 *Marketing Measurement and Analysis*	Graduate School of Business University of Texas at Austin	Robert P. Leone The University of Texas at Austin Dept. of Marketing Administration Austin, TX 78712
May 5-7, 1980	Washington, D.C. Shoreham Hotel	Donald Gross School of Engineering The George Washington Univ. Washington, D.C. 20052
*June 1980 *Marketing Science: An International Perspective*	Cergy, Pointoise, France ESSEC Sponsorship: TIMS/ORSA/MIT/ESSEC	Jean-Marie Choffray ESSEC Cergy, Pointoise, France Gary L. Lilien MIT Sloan School of Management Cambridge, MA 02139
†November 10-12, 1980	Colorado Springs, CO/ Broadmoor Hotel	R. Warren Langley 390 Buckeye Drive Colorado Springs, CO 80919
May 4-6, 1981	Toronto, Ont., Canada/ Four Seasons Sheraton	Murray Lister Strategic Policy Secretariat East Building 1201 Wilson Avenue Downsview, Ontario M3M IJ8 Canada
June 22-26, 1981 TIMS XXVI International Meeting	Cairo, Egypt Hilton Hotel Sponsorship: TIMS/UNIDO/IIASA	Mostafa El Agizy Exxon Corporation P.O. Box 153 Florham Park, NJ Ibrahim Badran President of Cairo University Giza, Cairo, Egypt
†October 12-14, 1981	Houston, TX Regency Hyatt House Hotel	James W. McFarland Quantitative Management Science 233 McElhinney University of Houston Houston, TX 77004
April 19-21, 1982	Detroit, MI/ Detroit Plaza	
Summer 1982	Lausanne, Switzerland	

**Special interest meeting.*
†*Incorrect dates previously listed.*