

AN OPTION-THEORETIC PREPAYMENT MODEL FOR MORTGAGES AND MORTGAGE-BACKED SECURITIES

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We introduce a new approach for modeling the prepayments of a mortgage pool and show how it can be used to value mortgage pools and agency mortgage-backed securities. We describe the full spectrum of refinancing behavior using a notion of refinancing efficiency. Our approach has two distinguishing features: (1) our primary focus is on understanding the market value of a mortgage, in contrast with standard models that strive (often unsuccessfully) to predict future cash flows, and (2) we use two separate yield curves, one for modeling mortgage cash flows and the other for MBS cash flows.

Keywords: Mortgage; prepayment model; mortgage-backed security; contingent claim; option; refinancing.

1. Introduction

Prepayments are the dominant consideration when valuing a mortgage-backed security (“MBS”). Most prepayment models in current use are econometric models that have been calibrated to historical prepayment data. Although the right to refinance a mortgage is widely recognized as an option, option-theoretic models are not currently used in prepayment modeling. Two reasons commonly given for this are that most homeowners do not exercise the option optimally and that option-based models are not able to explain observed prices of mortgage-backed securities.

In this paper we show that previous attempts at an option-based prepayment model lacked crucial features and that a rigorously constructed option-based model does in fact explain market prices of MBS very well. One outcome of our analysis is

that, contrary to conventional wisdom, MBS are in fact priced consistently with the assumption that most homeowners exercise their refinancing option near-optimally.

Mortgage-backed securities represent the largest sector of investment-grade fixed income products. The Lehman US Aggregate Bond Index, which includes only investment-grade products, has a mortgage component comprised of Ginnie Mae, Fannie Mae, and Freddie Mac pass-through securities. At 36% of the index as of September 2004 it is the largest component, exceeding the Treasury component that is about 24%. Mortgage-backed securities represent 48% of the spread products within the index.

Yet the current MBS valuation technology still consists of the same “reduced form” models, calibrated to historical data, introduced decades ago. These models do not use market-based arbitrage-free models, like those used for every other investment grade financial product. Reduced form models, which can predict only events that have historical precedent, have failed consistently over the last ten years, and for this reason they are no longer popular in other markets. At the same time, despite many attempts by both academics and practitioners to develop an option-theoretic model for MBS valuation, none appear to be in use today. The reason for this, in our opinion, is that previously proposed models have been either overly complex or mis-specified. Too much attention has been devoted to sophisticated term structure models, and too little to careful modeling of prepayments. The aim of this paper is to show that the standard valuation model for callable bonds, using a simple 1-factor short rate model, can be adapted to mortgages and reproduce MBS market prices.

The paper is organized as follows. In Sec. 2 we provide an overview of our approach. We introduce three distinct valuation models: for a Single Mortgage, an Unsecuritized Mortgage Pool, and an Mortgage-Backed Security. In Sec. 3 we provide a review of the literature — both practitioner and academic — on modeling mortgage-backed securities. Our prepayment model is presented in complete detail in Sec. 4, and in Sec. 5 we provide a rigorous mathematical formulation. In Sec. 6 we use our Unsecuritized Mortgage Pool model to analyze real mortgages and pools. We show that with the assumption that all homeowners refinance optimally, the model is able to fit observed prices of new mortgage pools by using realistic values for the turnover and curtailment rates, and homeowner and agency option-adjusted spreads. In Sec. 8 we show that using a simple and realistic distribution of prepayment behavior, our MBS model automatically accounts for the prepayment burnout effect and fits observed prices of seasoned MBS well.

2. Overview of the Model

A simple but important observation is that two different yield curves are needed, one to value mortgages from the perspective of the homeowner and the originator of the mortgage and the other to value an MBS from the perspective of the investor.

2.1. Single mortgage model

We begin by modeling a single mortgage and computing its value from the perspective of the homeowner. Although the end result of this computation is ultimately not needed, the computation process is itself important because it provides a model for when principal cash flows occur due to refinancing.

For convenience we shall restrict our attention to a standard 30-year fixed-rate mortgage; our analysis can be extended in a straightforward manner to other mortgage structures.

A standard fixed-rate mortgage is just an amortizing callable bond, sold at par by the homeowner to the mortgage originator or lender. If we assume for the moment that the homeowner will refinance the mortgage using an optimal strategy, then the mortgage can be valued using the following widely accepted method for valuing callable bonds:

- (1) Using a “bullet” mortgage par yield curve and volatility, build an interest rate lattice (using, say, the Black–Derman–Toy process [1]) calibrated to these inputs.
- (2) Starting with the final scheduled cash flows of the mortgage, compute the value of the mortgage for each node, working backwards through the lattice.
- (3) At each node compare the value of the existing mortgage with the value of a newly refinanced mortgage (assumed to be par plus the refinancing cost). If the latter is less, then the value of the existing mortgage is replaced by the value of the new one.

The obvious question is what yield curve and volatility to use for the homeowner. The yield curve used by our model is what we call the “Mortgagor Yield Curve”, which is a bullet yield curve implied by market-level mortgage rates. Our model specifies the Mortgagor Yield Curve as a fixed option-adjusted spread (“OAS”) off the swap curve. It also uses a constant volatility that is roughly consistent with the high-grade corporate and agency callable bond markets.

The model, however, needs to be extended because mortgages are not always refinanced using the optimal strategy implied by the option valuation algorithm. This type of extension is already needed for callable corporate bonds, because even corporate bonds are not always called optimally. Boyce and Kalotay [2] introduced the notion of “refunding efficiency” to parameterize the extent to which a corporation might want to call a bond non-optimally. We introduce in this paper a similar notion for mortgages, allowing us to model very simply non-optimal refinancing behavior.

The next key observation that needs to be modeled properly is the fact that mortgages are normally prepaid under two different circumstances: When the homeowner refinances at a lower rate or when the homeowner sells the house. As described above, we model the former at the level of an individual homeowner. We shall, however, model the latter, which we call “turnover”, only at the pool level. We

assume that when a homeowner considers whether to refinance or not, he contemplates holding the mortgage until maturity. In particular non-refinancing prepayments play no role in modeling the refinancing decision of a homeowner.^a

2.2. Unsecuritized mortgage pool model

The next step is to model a pool of mortgages from the perspective of an originator who has chosen to hold the pool as an asset. We will assume that a given pool contains only mortgages that have the same structure (say, 30-year fixed) and were all issued at approximately the same time. We also assume that all of the homeowners within the pool have similar credit profiles.

First, we must decide what yield curve to use for discounting the cash flows generated by the mortgage pool. In our model we use the Mortgagor Yield Curve (as defined above), which reflects both the funding cost for the homeowner and the rate of return received by the originator.

The next and most crucial challenge is modeling the cash flows generated by the pool. In our model we make a clear distinction between deterministic cash flows that are assumed to be insensitive to interest rate movements and cash flows that are interest rate dependent.

Given a pool of mortgages, prepayments will primarily occur for the following reasons: (1) sale of home, (2) default, (3) partial prepayment, and (3) refinancing. History shows that the first three are only weakly correlated with the level of interest rates and occur at relatively constant rates over time (if we ignore seasonality). We shall use the term “turnover” to mean prepayments due to home sales and defaults, and “curtailment” to mean partial prepayments made by the homeowner to shorten the life of the mortgage. We will model both using preset vectors of deterministic rates over the life of the mortgage pool.

Once the cash flows due to turnover and curtailment have been modeled and removed from the pool, we only need to model prepayments due to refinancing. At this point we bring in the single mortgage model described above. The twist is that we must assume that the pool contains a heterogeneous population of homeowners with different refinancing tendencies. We model this by dividing the pool into several buckets and assigning to each bucket a different refinancing behavior, ranging from leapers who refinance too early, to financial engineers who refinance optimally, to laggards who refinance too late. Each bucket is now valued from the perspective of the homeowner using the Single Mortgage Model; this tells us at which nodes in the lattice homeowners in each bucket will refinance and thus what cash flows will occur. By discounting these cash flows using the interest rate lattice (calibrated to the Mortgagor Yield Curve), we obtain the fair value of the pool.

^aThis assumption is easily refined. If we assume that the homeowner expects to remain in the house for only, say, five years, then we can simply value the mortgage as a 5-year balloon with a 30-year amortization schedule.

When this model is applied to a seasoned pool, the burnout effect (the phenomenon of seasoned pools prepaying slower than new pools) automatically appears as an intrinsic feature of the model. That burnout can be modeled in this way was first observed by Levin [22].

2.3. MBS model

Building a model for a pass-through security (the simplest form of MBS) is now rather straightforward. For convenience we assume that the underlying collateral consists of conforming 30-year fixed-rate mortgages and that the MBS have been guaranteed by either Fannie Mae or Freddie Mac. These two entities are government-sponsored enterprises. Their credit is not the same as the US government. Consequently, an investor in an MBS issued by one of these entities is exposed to credit risk.^b

First, observe that the Unsecuritized Mortgage Pool Model provides a framework for determining the mortgage cash flows. The MBS cash flows, in turn, consist of all principal cash flows and a predetermined portion of the interest cash flows of the mortgages. The balance of the interest cash flow is used for servicing costs and the guarantee fee.

It remains to decide how to discount the cash flows. The fundamental observation here is that the yield curve used to discount the MBS cash flows should be different from the one used to discount the unsecuritized pool cash flows. As noted before, the curve used for the latter is determined by market-level mortgage rates set by the originators. On the other hand, the MBS cash flows should be discounted using what we call the “MBS Yield Curve”, which is a bullet yield curve implied by market prices of MBS. The MBS Yield Curve reflects the yields demanded by MBS investors. We specify the MBS Yield Curve as an OAS off the swap curve.

We therefore need two OAS’s, one that gives the Mortgagor Yield Curve and the other the MBS Yield Curve. When valuing an MBS, the mortgage OAS is used to compute the non-deterministic cash flows, i.e., refinancings. Once the cash flows have been modeled, the MBS OAS is used for discounting the cash flows received by the MBS investor.

3. Traditional Approaches to MBS Valuation

There are two ways to determine the value of a fixed-income security whose cash flows are interest-rate-sensitive. One approach was to build an econometric model, fitted to historical data that attempts to predict the cash flows of the security. This approach has largely been abandoned, because efforts to accurately predict future cash flows have been futile. The more commonly used approach nowadays is to

^bIn contrast, an MBS guaranteed by Ginnie Mae carries the full faith and credit of the US government.

build an arbitrage-free model that is fitted to market prices of liquid instruments with credit similar to that of the security being priced. This approach places the focus on the market price of the security and the fundamental market factors that drive it. Today the latter is used universally for valuing virtually all interest-rate-sensitive securities, with one notable exception — MBS — that are still valued using econometric models.

3.1. *Econometric MBS valuation models*

The approach that predominates current prepayment modeling involves statistical analysis of historical prepayment data, which then serves as input into so-called econometric prepayment models. The problem with such models is that microeconomic factors driving the mortgage origination market can change dramatically. For example, prior to the mid 1990s, a rule of thumb was that mortgage rates would have to decline by 200 to 250 basis points (bps) below the homeowner's loan rate to justify refinancing. Since then there has been steady progress in whittling down the costs associated with refinancing. Today, some reports claim that even a 25 basis point decline would be sufficient to trigger a refinancing, particularly when lending programs allow the refinancing costs to be wrapped into the new mortgage loan. (This makes sense only if the homeowner does not have the cash to pay closing costs.) Moreover, the persistence of mortgage brokers and the wide dissemination of mortgage rate information have prodded homeowners into identifying opportunities to refinance. Econometric prepayment models have consistently failed during fast prepayment periods. While MBS analysts continually update their prepayment models, their models will always lag behind shifts in the microeconomic structure of the mortgage market.

3.2. *Option-theoretic MBS models*

Many option-theoretic approaches to valuing MBS have been proposed, primarily in the academic but also in the practitioner literature. On the academic side, the first to introduce an option-theoretic MBS model was Kenneth Dunn, initially in his doctoral dissertation at Purdue and then in a series of papers with McConnell [8, 9]. The Dunn–McConnell model assumes optimal refinancing decisions and exogenous reasons for other prepayments.

The model was extended by Timmis [27], Dunn and Spatt [10], and Johnston and Van Drunen [18] by incorporating transaction cost or other frictions to explain non-optimal refinancing decisions. The main shortcoming of these models is that all homeowners are assumed to behave identically and optimally under specified economic conditions, implying that all refinancings occur simultaneously in a given pool.

On the practitioner side, Davidson and Hershovitz [11] when at Merrill Lynch developed a “threshold refinancing pricing model”. In their model, a mortgage pool is heterogeneous, and non-optimal refinancing is explained by the premise that

different homeowners face different transaction costs. Similar heterogeneous pool models have also been proposed and analyzed by practitioners such as Klotz and Shapiro [21], Chen [3, 4], Cheyette [5], and Levin [22], and in the academic literature by Stanton [25], Deng [6], Deng, Quigley, and Van Order [7], and Kau and Slawson [20].

The Davidson–Hershovitz model was implemented at Merrill Lynch, but never gained wide acceptance and was eventually abandoned. To the best of our knowledge, no other commercial vendor or dealer firm ever implemented an option-based MBS model, despite the migration of several leading financial theorists from academia to leading Wall Street firms.^c

The failure of past option-based models has been due to their inability to explain and match market MBS prices. The key reason for this lies in the modeling of the refinancing decision. These models all use the wrong yield curve, either the Treasury or swap curve, to model how the mortgagor decides to refinance. Since these curves do not accurately reflect the actual cost of funds for a homeowner, the models can be calibrated to realistic mortgage rates and MBS prices only by using ad hoc parameters (such as transaction costs) set to artificial values. For example, a high coupon MBS tends to have a higher than expected price, due to homeowners in the pool who should refinance but do not. The option-based models described above do not model this behavior directly and do it indirectly by assigning artificially large transaction costs to a fraction of the homeowners in the mortgage pool. The need to use ad hoc unrealistic parameter values leads to a model that does not behave properly as the interest rate environment changes. For this reason previously proposed option-based models, despite their advantages over reduced form models, have never been able to establish any foothold among practitioners.

4. An Option-Based Prepayment Model for Mortgages

The model presented in this paper is similar in some regards to earlier option-theoretic models, but differs in crucial aspects. It is similar, because the model of refinancing behavior is based on an optimal option exercise strategy. We also model heterogeneity by breaking the mortgage pool up into buckets and assuming that each bucket represents different refinancing behavior.

^cFor example, in the 1980s, Goldman Sachs, a major dealer in MBS, hired Richard Roll from UCLA and Scott Richard from Carnegie Mellon to manage their MBS research group responsible for the firm's prepayment and valuation models (see [23]). The models that they developed for the firm were not based on the models that dominated the academic literature. Moreover, several prominent academics moved to the money management area specializing in MBS. Two notable examples are Kenneth Dunn from Carnegie Mellon who managed funds for Miller, Anderson & Sherrerd and Douglas Breeden who began an MBS advisory firm as well as starting a savings and loan association. To the best of our knowledge, the academic models proposed in the literature were not implemented by their firms.

The crucial features distinguishing our model from the others are the following:

- Mortgage cash flows and MBS cash flows are discounted using different yield curves. As far as we know, no previous model does this.
- It classifies prepayments as two different types, and models each differently. The first type, turnover, is assumed to be independent of interest rates, and the second, refinancings, is assumed to depend on interest rates.
- Refinancing behavior is not modeled in terms of transaction costs but in terms of an “imputed coupon”, which is defined later in this section.

We have already discussed why it is necessary to use two different yield curves. Note that it also immediately affords a simple and natural way to model the credit profile and the credit impairment of the homeowner (as an OAS). Our model handles real transaction costs (and therefore the “lifetime refinancing cost”, as studied in the academic literature) in a straightforward and realistic manner. In addition, our model provides a simple means of parameterizing the full range of possible sub-optimal refinancing tendencies (“leapers” to “laggards”).

Existing prepayments models are based upon overly complex descriptions of prepayments, as well as overly sophisticated interest rate models. They all have many input parameters to set or calibrate and run into the danger of overfitting. Our aim is a parsimonious prepayment model that uses the simplest possible mechanisms to account for all crucial factors that drive the price of an MBS. We will show that by properly incorporating transaction costs, our model, using only a few input parameters set at reasonable and realistic values, is able to reproduce the market prices of new mortgage pools and pass-through MBS and how they change as market conditions change.

We will model the prepayment process of the mortgagor in considerable detail. Once we have a simple but realistic formulation of the prepayment process, we will be able to understand how “turning the knobs” affects cash flows and value.

4.1. *Model of turnover*

We distinguish between interest-rate driven refinancings and all other prepayments. We assume that the sole purpose of refinancings is to reduce interest expense. Using the term more broadly than is customary, we will refer to all other prepayments as turnover. The primary cause of turnover is sale of property. Other reasons are cashouts, defaults, and destruction of the property. While the distinction between refinancing and turnover is admittedly somewhat blurry — home sales may in fact depend on interest rates — for modeling purposes we consider our approach adequate.

We describe turnover by a vector of monthly prepayment rates (referred to as “speeds”) over the legal life of the mortgages. In the examples that we present in this paper, we assume that this vector is a percentage of the industry-standard Public Securities Association (PSA) prepayment benchmark. This benchmark, developed

in the early 1980s, is used for all types of mortgage designs for agency mortgage-backed securities issued by Ginnie Mae, Fannie Mae, and Freddie Mac.^d According to the PSA prepayment benchmark, which is a vector of conditional prepayment rates (CPRs), after 30 months approximately 6% of the outstanding mortgages prepay annually. For the first 30 months, the CPR increases linearly by 0.2% per month. So if the turnover rate is 50% of the PSA (50% CPR) from month 30 on, approximately 3% of the mortgages prepay annually, excluding refinancings.

According to expert opinion, turnover is somewhere between 75% PSA and a 100% PSA. Data from periods of high interest rates — and therefore low rates of economical refinancing — support this opinion. Based on historical experience, a turnover rate of 50% PSA is too low, while 150% PSA is too high.^e We will demonstrate how our model can determine the market-implied turnover rate. Our results are in good agreement with expert opinion and historical experience, i.e., we find that 50% PSA is too low, and 150% PSA is too high. In subsequent examples we keep the turnover rate at 75% PSA.

We estimate the baseline rate assuming that the interest rate of new mortgages incorporates/reflects the market's expectation of turnover. We also note that:

- (1) While turnover is a source of uncertainty, it is not necessarily detrimental to investors. It reduces the average life of the mortgage pool and, occurs even when the mortgagor's loan rate is below the market rate.
- (2) In contrast to turnover, refinancings benefit homeowners strictly at the expense of the investors. Therefore the cost of the prepayment option, namely the excess interest above the optionless rate, should be attributed primarily to refinancing risk rather than turnover risk.
- (3) Turnover reduces the average life of mortgages, and thus the refinancing risk. The higher the expected turnover, the less the market charges for the refinancing option.

4.2. Model of refinancing behavior

From the perspective of the mortgagor, the risks entailed in the refinancing decision are obvious: *ex post*, refinancing is seen as premature if rates continue to decline, while waiting is perceived to be a mistake if rates rise. There are rigorous option-based valuation tools available to assist borrowers with the timing decision. Many

^dGinnie Mae MBS are referred to as agency MBS and are backed by the full faith and credit of the US government. Fannie Mae and Freddie Mae issued MBS are referred to as "conventional MBS" and expose the investor to credit risk since these two entities are government sponsored enterprises. For non-agency mortgage-backed securities and for home equity loan- and manufactured housing loan-backed securities in the asset-backed securities market, the current practice is to describe the prepayment behavior of a pool of loans in terms of the issuer's prospectus prepayment curve (PPC) rather than in terms of the PSA prepayment benchmark.

^eThere is ample empirical evidence that for low coupon agency pass-through securities turnover is the dominant factor explaining prepayments. See Hu [17] in particular.

corporate and municipal bond issuers routinely employ the “efficiency” approach described below in their refunding decisions.^f

Why the concept of refinancing efficiency has been absent from the MBS literature is an enigma.

The basic idea is to treat the mortgagor’s right to refinance as a formal call option, exercisable at any time at par. Given the prevailing mortgage rates and a market-based interest rate volatility, the mortgagor can determine the value of the refinancing option and compare it to the attainable savings (expressed in present value terms). The ratio of savings to option value is the so-called “refinancing efficiency”. Although refinancing efficiency cannot exceed 100%, it will reach 100% if rates are sufficiently low. At 100% efficiency the expected cost of waiting for interest rates to decline further exceeds the cost of the new mortgage. Financially sophisticated borrowers will “pull the trigger” when the refinancing efficiency reaches 100%.

Calculation of savings and option value requires an optionless yield curve. While optionless borrowing rates are readily available to institutional borrowers, residential mortgage rates are always for immediately callable loans. We will revisit this consideration below.

Admittedly, few homeowners possess the financial sophistication described above. We will refer to those who do as “financial engineers”. Most homeowners refinance too early (at an refinancing efficiency less than 100% of the option value) or too late (they continue waiting after refinancing efficiency has reached 100%). We will refer to early refiners as “leapers” and those who act late as “laggards”. Together, financial engineers, leapers, and laggards span the entire spectrum of refinancing behavior.

We will establish in Sec. 6 that a rational leaper cannot refinance much sooner than a financial engineer; doing so would actually result in a loss, rather than in savings. For this reason our focus will be on laggards, rather than on leapers. We now provide a formal definition for leapers and laggards.

Our parameterization is a natural extension of the definition of the financial engineer. We characterize refinancing behavior by assigning the mortgagor an “imputed coupon”. The mortgagor will refinance whenever a financial engineer would refinance a maturity-matched mortgage with the imputed coupon. For example, consider two 7% mortgagors who refinance sub-optimally — one does so when a financial engineer refinances a 6% and the other when a financial engineer refinances a $7\frac{1}{2}\%$ mortgage. Because the former’s imputed coupon is 6% and actual is 7%, we refer to him as a 1% laggard (who refinances late). Similarly, the one with an imputed coupon of $7\frac{1}{2}\%$ is a $\frac{1}{2}\%$ leaper (or equivalently a $-\frac{1}{2}\%$ laggard), who refinances early. We make the following related assumptions:

- (1) The turnover rate is uniform across all types of mortgagors, be they leapers, laggards, or financial engineers.

^fSee Boyce and Kalotay [2] and Howard and Kalotay [12]. For a review of the underlying theory, see Kalotay, Williams and Fabozzi [19].

- (2) Migration over time across behavioral types is not allowed: once a laggard, always a laggard.
- (3) The refinancing decision of a financial engineer does not depend on the expected turnover of the pool. In other words, the cash flow savings and the option value are calculated assuming the full remaining term of the mortgage.

4.3. Burnout

Prepayment burnout, or simply burnout, refers to the observed slowdown of interest-rate driven prepayments following periods of intensive refinancings. It is attributed to changing distribution of the pool: The most aggressive mortgagors (leapers) are the first to refinance, leaving behind the slower reacting laggards.

Conventional MBS analysis handles burnout by changing the parameters of the prepayment function. One possible refinement is to partition the pool by prepayment speeds, so that the earliest prepayments will be attributed to the fastest sector (see [22]).

From the “pool factor” — the ratio of the mortgage pool’s remaining principal balance outstanding to the mortgage pool’s original principal balance — we can infer the extent of prepayments. Contractual (i.e., regularly scheduled) amortization determines at any given time the maximum possible value of the pool factor, assuming no prepayments at all (i.e., 0% PSA). Any difference between this maximum value and the actual value is due to prepayments. Although conventional prepayment models do not explicitly distinguish between turnover and refinancing *ex post*, low pool factors for high-coupon mortgage pools are understood to be primarily due to refinancings.

In our model, burnout requires no special treatment. The assumed turnover determines a baseline value for the pool factor. Any difference between the baseline pool factor and the actual pool factor is attributed to refinancings. Because the model specifies the order in which mortgagors refinance (leapers first, followed by financial engineers, and then by laggards), the pool factor unambiguously determines who has left and who still remains in the mortgage pool. In statistical terms, the pool factor determines the conditional distribution of leapers and laggards, given an initial distribution.

Therefore burnout is a natural consequence of our model. Given two otherwise identical mortgage pools, the one with the smaller pool factor will automatically prepay more slowly.

5. Mathematical Formulation

5.1. Interest rate dynamics

Although we use a 1-factor constant volatility lognormal short rate model [1], any stochastic term structure model can be used with the prepayment model described in this paper.

We model the dynamics of the swap curve using a constant volatility lognormal short term interest rate model, whose stochastic differential equation is given by

$$dr = ar dt + \sigma r dZ ,$$

where σ is the constant volatility, r is the short term interest rate, a is a deterministic function of time t , and Z is the standard Brownian process. We use the variant proposed by Sandmann and Sondermann [24], where r is a semiannually compounded rate instead of a continuously compounded rate. The model is calibrated to a spot curve obtained by stripping the USD swap curve (LIBOR rates, Eurodollar futures, and par swap rates).

In particular, the price at time t of a hypothetical zero-coupon LIBOR bond that pays \$1 at time T is given by

$$D(t, T, Z) = E \left[\exp \left(-2 \int_t^T \ln \left(1 + \frac{r(\tau, Z(\tau))}{2} \right) dZ(\tau) \right) \middle| Z(t) = Z \right] ,$$

where the expectation E is conditional on $Z(t) = Z$ at time t .

The dynamics of mortgage and MBS yields are modeled using the standard option-adjusted spread (OAS) approach described below.

Given an option-adjusted spread s_{mtge} for the mortgagor, we model the dynamics of optionless mortgage rates by setting the “short term mortgage rate” to

$$r_{\text{mtge}} = r + s_{\text{mtge}} ,$$

where the spread s_{mtge} is either a constant or a given deterministic function of t . In particular, the value at time t of a “zero coupon mortgage” that pays \$1 at time T is given by

$$D_{\text{mtge}}(t, T, Z) = E \left[\exp \left(-2 \int_t^T \ln \left(1 + \frac{r_{\text{mtge}}(\tau, Z(\tau))}{2} \right) dZ(\tau) \right) \middle| Z(t) = Z \right] .$$

Similarly, given a spread s_{mbs} for the MBS cash flows, we model the dynamics of the interest rates used for discounting MBS cash flows by setting the “short term MBS rate” to

$$r_{\text{mbs}} = r + s_{\text{mbs}} ,$$

where the spread s_{mbs} is either a constant or a given deterministic function of t . In particular, the value at time t of a “zero coupon MBS” that pays \$1 at time T is given by

$$D_{\text{mbs}}(t, T, Z) = E \left[\exp \left(-2 \int_t^T \ln \left(1 + \frac{r_{\text{mbs}}(\tau)}{2} \right) dZ(\tau) \right) \middle| Z(t) = Z \right] .$$

5.2. Valuing a mortgage

A mortgage can be refinanced at any time, but for convenience we will assume that it is refinanced only on cash flow dates. We will assume for convenience that there is no notification period for refinancing. We value the mortgage as an amortizing callable bond.

Let t_0 denote today. Let t_1, \dots, t_N denote the cash flow dates of a mortgage and P_1, \dots, P_N denote the corresponding principal cash flows, as a fraction of the original mortgage principal. Let m denote the (periodic) fixed mortgage rate. Given $0 \leq k \leq N - 1$, let R_k denote the principal remaining (after the coupon payment) at time t_k . The coupon payment at time t_k is given by $C_k = P_k + mR_{k-1}$.

We assume that the refinancing cost at any time is a fixed fraction c of the remaining principal. Denote the effective strike at time t_k by

$$S_k = R_k(1 + c).$$

Given a mortgage rate m , let $V_m(t_k, Z(t_k))$ denote the value (as a fraction of original principal and after the coupon payment) at time t_k of a mortgage held by a “financial engineer”. Let $W_m(t_k, Z(t_k))$ denote the value of the mortgage at time t_k , if it is not refinanced at time t_k . They are given recursively for $k = 0, \dots, N$ as follows.

$$V_m(t_N, Z(t_N)) = W_m(t_N, Z(t_N)) = 0$$

$$W_m(t_k, Z(t_k)) = D_{\text{mtge}}(t_k, t_{k+1}, Z(t_k))(P_{k+1} + mR_k + E[V_m(t_{k+1}, \cdot) | Z(t_k)])$$

$$V_m(t_k, Z(t_k)) = \min(S_k, W_m(t_k, Z(t_k))).$$

The value $V_{m,\ell}(t_k)$ of a mortgage held by a homeowner with a nonzero laggard spread ℓ is given for $k = 0, \dots, N - 1$ by

$$V_{m,\ell}(t_N, Z(t_N)) = 0$$

$$V_{m,\ell}(t_k, Z(t_k)) = \begin{cases} S_k, & \text{if } S_k < V_{m-\ell}(t_k, Z(t_k)) \\ W_m(t_k, Z(t_k)), & \text{otherwise.} \end{cases}$$

Observe that the decision whether to refinance or not is made on the basis of a mortgage with a rate of $m - \ell$, even though the actual mortgage rate is m .

5.3. Valuing an MBS

For convenience, we assume that there is no time lag between mortgage payments and MBS payments. In practice there is, of course, at least a 14 day payment lag that must be accounted for properly in the implementation of the prepayment model.

Let Π_1, \dots, Π_N denote the principal payments that occur at times t_1, \dots, t_N of a pass-through MBS, given as a fraction of the original principal. We assume that each Π_k contains the scheduled principal cash flows, as well as principal payments

due to turnover, curtailment, and defaults. Let S_1, \dots, S_N denote the principal remaining at times t_1, \dots, t_N , as a fraction of the original principal.

Let m denote the (periodic) weighted average mortgage rate, and M denote the (periodic) MBS coupon rate. If we assume that the entire pool consists of homeowners with a laggard spread of ℓ , then the value of the pool would be given recursively by

$$\begin{aligned}
 &V_{M,m,\ell}(t_N, Z(t_N)) = 0 \\
 &V_{M,m,\ell}(t_k, Z(t_k)) = \begin{cases} R_k & \text{if } S_k < V_{m-\ell}(t_k, Z(t_k)) \\ U_{M,m,\ell}(t_k, Z(t_k)) & \text{otherwise,} \end{cases}
 \end{aligned}$$

where

$$U_{M,m,\ell}(t_k, Z(t_k)) = D_{\text{mbs}}(t_k, t_{k+1}, Z(t_k))(\Pi_{k+1} + MS_k + E[V_{M,m,\ell}(t_{k+1}, \cdot) | Z(t_k)]).$$

Observe that the decision of whether refinancing occurs depends on the mortgage value and not the MBS value.

If $p(\ell)d\ell$ is the laggard spread distribution for the mortgage at time t_0 , then the value of the MBS at time t_0 is given by

$$V_{M,m}(t_0) = \int_0^\infty V_{M,m,\ell}(t_0)p(\ell)d\ell.$$

5.4. The laggard spread distribution for a seasoned MBS

Let f_0 denote the factor of a mortgage pool that would be expected from scheduled prepayments, turnover, curtailment, and defaults, if there were no refinancings at all. Let f denote the actual factor. Therefore, $1 - f$ of the original principal has prepaid, and $f_0 - f$ represents the prepayments due to refinancings.

Let $p_0(\ell)d\ell$ represent the laggard spread distribution for the mortgage pool at $t = 0$. Then the laggard spread distribution $p(\ell)d\ell$ for the pool at time t_0 is obtained from the original distribution p_0 and the factors f_0 and f by

$$p(\ell) = \begin{cases} 0 & \text{if } \ell > \ell_0 \\ p_0(\ell)/p & \text{otherwise,} \end{cases}$$

where p and ℓ_0 are given by

$$\begin{aligned}
 \int_0^{\ell_0} p_0(\ell)d\ell &= \frac{f_0 - f}{f_0} \\
 p &= \int_{\ell_0}^\infty p_0(\ell)d\ell.
 \end{aligned}$$

In other words, refinancings cause the least laggard homeowners to exit the pool first, leaving the more laggard homeowners. This therefore models the burnout effect observed in mortgage pools.

6. Valuation of Mortgages

In this critical section we will present in considerable detail our option-based valuation method of mortgages and illustrate it with several examples. After we have mastered the analytics of mortgages, extending the technology to MBS will be straightforward.

The section begins with a discussion of the term structure of optionless mortgage rates. While optionless these rates are readily observable in virtually all other sectors of the credit markets, in the realm of residential mortgages they are virtually non-existent. One of our principal objectives is to determine, in terms of basis points, the market cost of the prepayment option. By carefully distinguishing between turnover and refinancings, we establish that turnover actually reduces the cost of the option. As pointed out previously, the analytical tools that we employ are well known by capital markets participants and widely used in the structuring and refunding of callable corporate and municipal bonds.

6.1. *The term structure of mortgage rates*

Arbitrage-free valuation of a fixed-income instrument requires as an input an optionless yield curve. This curve can be converted into spot and discount rates by bootstrapping, or into a lattice to value an instrument with an embedded option (see [19]).

Because conventional residential mortgages are immediately prepayable, we cannot observe an optionless yield curve directly. In addition, standard mortgages rates are for amortizing structures rather than bullets. In this section we will analyze mortgages assuming that an optionless yield curve is actually observable. Later on we will discuss how this yield curve can be inferred from prevailing mortgage rates.

Let us first discuss the effect of servicing cost. The cash flows received by an investor must reflect the net servicing cost; the higher the servicing cost is the lower the value of the mortgage. We also note that servicing cost does not reduce the cash flows paid by the mortgagor.

Given the annual servicing cost, we can determine the fair value of an optionless mortgage with a specified interest rate and maturity. Or, as shown below, we can determine the interest rate on a new optionless mortgage that sells at par. Figure 1 shows our assumed optionless bullet mortgage rates. This mortgage curve is 80 bps above the fixed side of a maturity-matched LIBOR swap curve. Note that the yield curve is steeply upward sloping and that the 30-year “bullet” (i.e., non-amortizing) rate is 5.60%.

Figure 2 shows the fair interest rate of optionless amortizing mortgages of various maturities. For example, in the absence of servicing cost the fair rate on a 30-year level-pay mortgage is 5.15%, 45 bps below the 30-year bullet rate. The reason for the difference is the upward sloping yield-curve — the average life of a 30-year mortgage is only 18.75 years. In general, the shorter the final maturity the lower is the rate. Also shown in Fig. 2 are fair mortgage rates incorporating service costs.

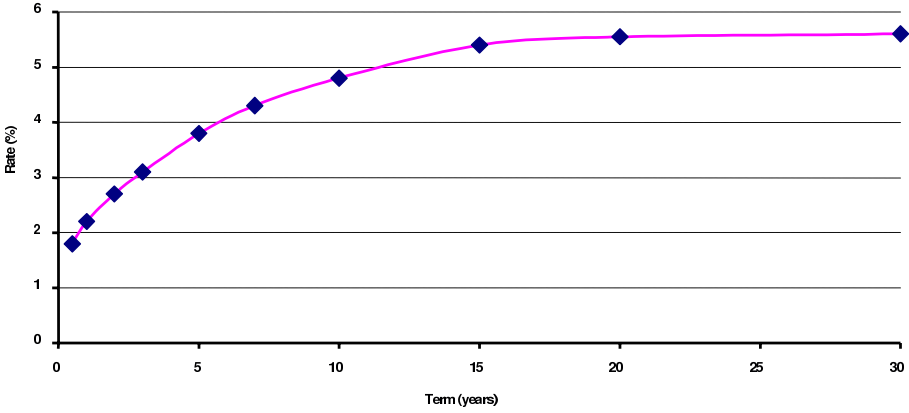


Fig. 1. Optionless “bullet” mortgage yield curve.

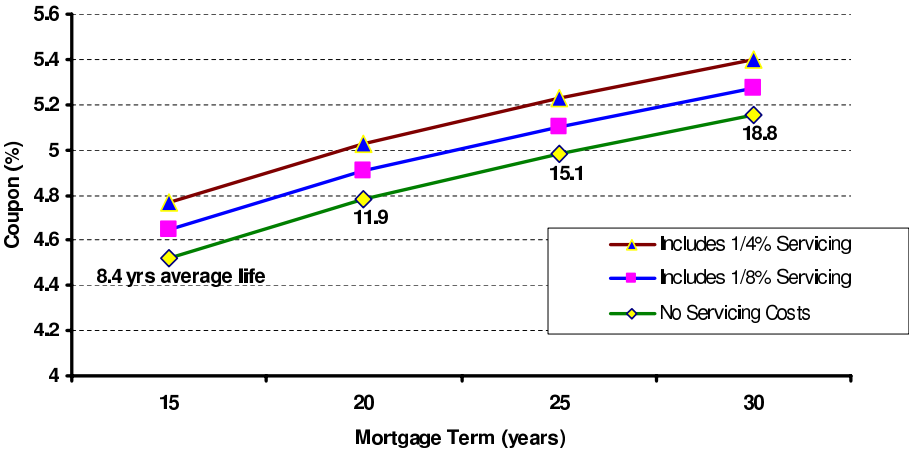


Fig. 2. Fair coupon on non-prepayable mortgages.

The servicing cost is essentially additive. For example, if the annual servicing cost is 0.25%, the fair mortgage rate increases from 5.15% to 5.40%.

Expert estimates of the annual servicing cost vary between $\frac{1}{16}\%$ and $\frac{1}{4}\%$. In the examples below, we assume that the cost is $\frac{1}{8}\%$, resulting in a rate of roughly 5.28% for a new 30-year mortgage.

6.2. How turnover affects mortgage rates

We calculated above the fair interest rate of optionless mortgages. We now consider hypothetical mortgages that can be prepaid but cannot be refinanced. This notion is the exact analogue of the familiar “callable but not refundable” feature of corporate bonds. But while for bonds this notion is relatively unimportant, for mortgages it is extremely significant because turnover is a principal source of prepayments.

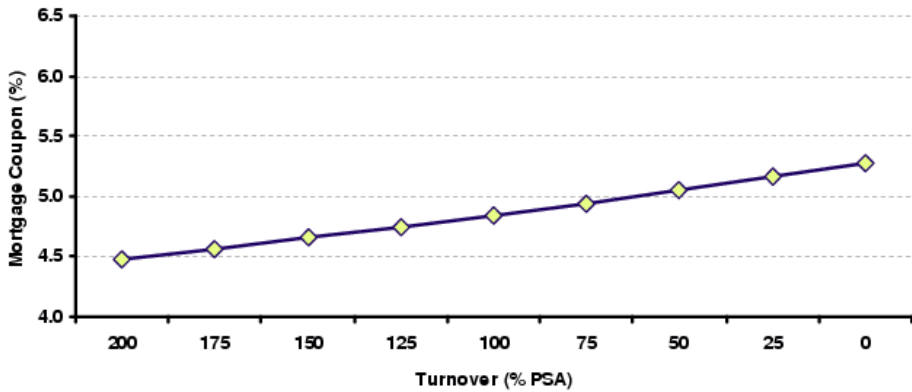


Fig. 3. Fair coupon of 30-yr prepayable but non-refinanceable mortgage (including 1/8% servicing cost).

Consider a large pool of new 30-year mortgages and assume that its annual turnover can be accurately predicted. We parametrize turnover as a multiple of the PSA speed; in particular 0% depicts no prepayments at all. The annual servicing cost is assumed to be 0.125%. Figure 3 shows how turnover affects the rate of a new 30-year non-refinanceable mortgage. As in Fig. 2, because of the upward-sloping yield curve the higher the turnover (i.e., the shorter the average life) the lower will be the mortgage interest rate.

6.3. The refinancing decision

Next we would like to analyze the effect of the refinancing option. But first we need to model how mortgagors can approach the refinancing decision using the notion of refunding efficiency.

For illustration, we will consider mortgages with 25 years remaining to maturity. Because we report all values as a percentage of the outstanding face amount, the dollar size of the mortgage is irrelevant. We assume that the refinancing expenses amount to 1%.

For valuation purposes the mortgagor and the investor should use the same yield curve (and associated lattice), because they are looking at essentially the same cash flows. Note, however, that for an insured mortgage — the type of mortgage included in a Ginnie Mae pool backing an MBS — this argument would not apply.

The textbook approach for determining the savings from refinancing (see, for example, [14]) assumes that the new mortgage is optionless and matches the amortization schedule of the outstanding one, for the remaining 25 years. In practice the terms tend to be mismatched, and the new mortgage may be a full 30-year loan that is immediately repayable. We will revisit this issue below.

Consider a 6% mortgage with 25 years to maturity; its remaining average life is about 15.5 years. The rate of a matching refinancing mortgage turns out to be

5.41%. The savings is equal to the difference between the present values of the existing mortgage and the new one. The amount saved (net of refinancing cost) would be 5.05% of the outstanding principal amount. On the other hand, at 16% volatility the option value (accounting for current and future refinancing costs) is 5.675% of the outstanding principal amount. The resulting refinancing efficiency is their ratio, 89.2%. For a 6.25% mortgage the savings is 7.592%, the corresponding option value is 7.63%, and the refunding efficiency is 99.5%. Figure 4 displays how the refinancing efficiency responds to changes of the mortgage yield curve. For example, the yield curve would have to decline 23 bps, corresponding to a 5.18% refinancing rate, in order that the efficiency of the 6% mortgage reach 100%.

We conclude that a financial engineer would refinance a 6% mortgage with 25 years to maturity if he could obtain a matching 5.18% mortgage (an annual saving of 82 bps before transaction cost), while the target rate for a 6% mortgage is about 5.40% (a saving of 85 bps).

We note that efficiency depends not only on the refinancing rate but also on the shape of the yield curve and the interest rate volatility, here assumed to be 16%. At a higher volatility the option value would increase and therefore the 100% efficiency level would require a lower rate. It is worth noting that standard reduced form models do not consider the effect of interest rate volatility on prepayment speed.

As mentioned above, in practice the new mortgage does not match the maturity structure of the outstanding one and it is also repayable. How does the refinancing efficiency approach cope with these considerations?

The basic idea is that as long as the refinancing mortgage is fairly priced, its precise structure is irrelevant; its maturity and coupon structure can be arbitrary. For example, it is possible to determine the savings from refunding a 25-year fixed-rate mortgage with a 30-year adjustable-rate mortgage. Returning to the example of the 6% mortgage with 25 years left to maturity, we saw that it should be refinanced

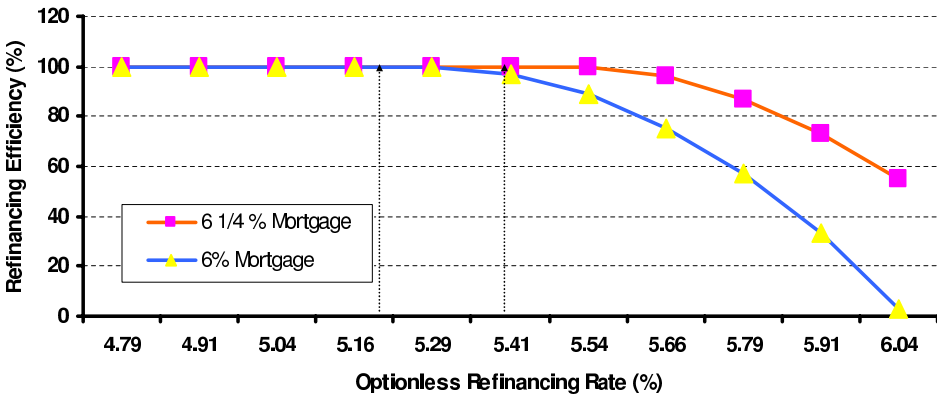


Fig. 4. Refinancing efficiency for 25-year mortgages.

if it is possible to obtain a matching 5.18% 25-year mortgage. But the mortgagor could also opt for a 30-year non-refinanceable mortgage with a slightly higher rate (say 5.22%) or even a 30-year refinanceable rate (say 5.62%, more about this later). The practical problem is that a mismatch introduces interest rate risk, and therefore the savings are not guaranteed. For example, if interest rates rise, refinancing a fixed-rate mortgage with one that floats can result in a loss, rather than in savings. Note that this problem does not arise when the mortgages are matched, because the periodic cash flow savings are known.

The refinanceability of the new mortgage poses a similar problem. Because a refinanceable mortgage bears a coupon higher than an otherwise identical optionless mortgage, the nominal cash flow savings are lower. On the other hand, there is the potential of additional savings should rates continue to decline further. The critical question is whether or not the refinancing feature is fairly priced by the market.

Adjusting both savings and option value for possible mispricing of the new mortgage is straightforward. If such mispricing favors the borrower, the adjusted refinancing efficiency will be higher, and this should advance the timing of refinancing.

It is interesting to speculate whether or not the refinancing option is fairly priced in a new mortgage. The answer depends, among other factors, on the average efficiency at which mortgagors refinance. This question, however, is beyond the scope of this paper. For the present purposes, we assume that new mortgages are fairly priced. At the same time, we observe that our approach allows us to incorporate systematic mispricing of new mortgages, along the lines discussed above.

6.4. The market cost of the refinancing option

Let us now extend the valuation method described above to determine how many bps the market charges for the refinancing option on a new 30-year mortgage.

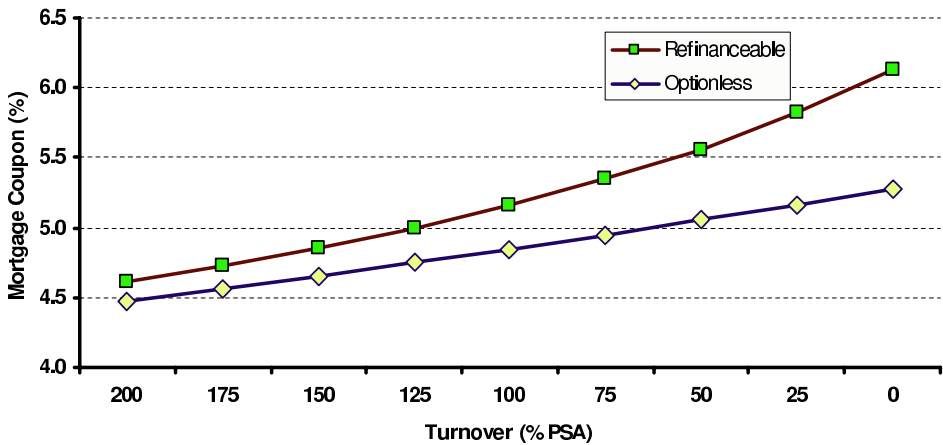


Fig. 5. Fair coupon for 30-year mortgages (including 1% refinancing cost).

Figure 3 serves as a reference: it shows the non-refinanceable rate at an assumed turnover.

Figure 5 displays the basic results. The critical assumptions are that the interest rate volatility is 16%, the refinancing cost is 1%, and the refinancing decisions are optimal (i.e., every mortgagor is a financial engineer). Each of these assumptions can be easily changed. For now, let us examine the implications of our current findings.

Figure 6 shows the cost of the refinancing option as a function of turnover, obtained by subtracting the optionless rates from the refinanceable rates. We observe that the lower the turnover the more the market charges for the refinancing option: for example, at a turnover of 0% PSA, the cost would be 86 bps, while at 150 PSA it is only 20 bps. As indicated earlier we will assume that the expected turnover rate for a new mortgage pool is 75% of PSA; at that rate the current estimated cost of the refinancing option would be roughly 40 bps. This result seems reasonable and it provides additional justification for using the 75% PSA assumption for turnover.

The above results are based on the assumption that every mortgagor is a financial engineer. We could easily modify this assumption to a specified leaper/laggard behavior, or more generally assume that the initial pool can be represented by some distribution of leapers and laggards. In the interest of brevity we state without providing results that the cost of the refinancing option of new mortgages depends primarily on turnover, and only weakly on leaper/laggard distribution. At the same time we observe that as the pool ages it becomes skewed toward laggards, and therefore the leaper/laggard distribution is critical in valuing seasoned pools.

In summary, we have shown that a financial engineer would refinance a long-term mortgage with a non-refinanceable mortgage, if he could save about 85 bps. But the market offers only refinanceable mortgages at roughly 40 basis points higher.

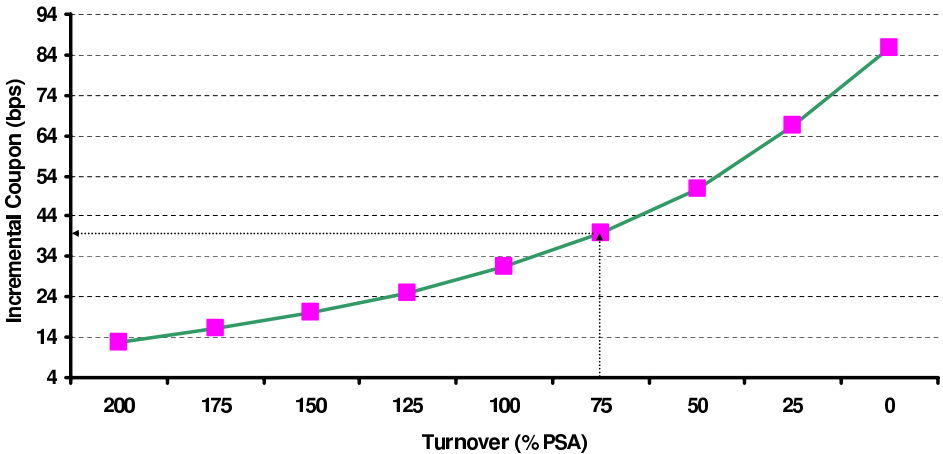


Fig. 6. Cost of refinancing option.

Therefore a financial engineer will refinance a long-term mortgage when the market rate is about $85 - 40 = 45$ bps below the rate of the outstanding mortgage balance.

6.5. Seasoned mortgages and tranches

Let us turn to seasoned mortgages. We assume that the mortgagor is a financial engineer and, in order to demonstrate certain points, that the servicing cost is nil. Figure 7 shows the values of pools of uninsured 30-year mortgages over a wide range of coupons. In anticipation of valuing mortgage derivatives products (collateralized mortgage obligations and mortgages strips-, interest interest-only and principal-only securities), Fig. 7 also displays the interest and principal components.

We observe that the value initially increases, reaching a peak of roughly 101 at a 6% coupon. As the coupon further increases, the value declines to slightly above 100. Let us discuss the reasons for this phenomenon.

Consider the right side of the figure. Because a financial engineer will refinance a mortgage whose coupon is very high without delay, we would expect the value of such mortgage to be very close to 100. The slight premium observed in the graph is due to the additional interest received by the investor during the period from when the homeowner notifies the bank of his intent to pay off the mortgage to the actual refinancing date.

It is less intuitive, however, how the value of a mortgage can exceed 101 if the refinancing option is optimally exercised. The 1% refinancing cost is clearly insufficient to explain this phenomenon. The fundamental reason is that the mortgagor’s refinancing decision disregards turnover. In essence, the mortgagor refinances only when it would make sense to do so with a maturity-matched mortgage, rather than with a mortgage that recognizes that for some reason unrelated to interest rates the mortgage may have to be prepaid. Because when he refinances a mortgagor does not plan on moving in the foreseeable future, such a “myopic” decision policy seems

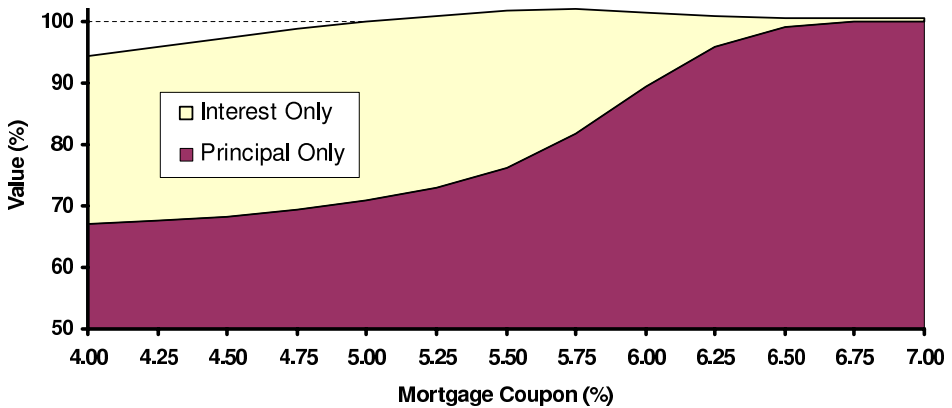


Fig. 7. Value of principal and interest components of uninsured mortgage pools.

realistic. The effect of turnover on the value of the mortgage is extenuated when the yield curve is steeply upward sloping, as is the case here. While financial engineer mortgagors are patiently waiting for interest rates to decline to a level where refinancing is optimal, some of them end up prepaying for unrelated reasons. This explains why the fair value of a mortgage can exceed par even under the assumption of optimal option exercise.

7. A Closer Look at Leapers and Laggards

7.1. *The range of leaper/laggard spreads*

Although our ultimate goal is to value MBS, for the time being we will continue to focus on the underlying unsecuritized mortgage pool. The differences between the MBS and the pool are twofold. First, only a specified portion of the mortgage interest is passed through to the MBS holder, and second, because of credit enhancement and liquidity considerations the MBS cash flows are preferable to the mortgage cash flows — and hence discounted at a lower rate. We will re-examine these issues in the valuation of MBS.

As discussed above, implementation of our methodology requires a user-specified turnover rate and leaper/laggard distribution. We have provided heuristic evidence to the reasonableness of using 75% of PSA for the turnover rate. Assuming this and that the mortgagor is a financial engineer, we have calculated the fair interest rate of a new refinanceable mortgage.

We now relax the assumption that every mortgagor is a financial engineer, and consider how a heterogeneous pool affects value. We provided earlier a rigorous definition for leapers and laggards: using financial engineers as a point of reference we specified behavior by a spread relative to the financial engineer. Next we will establish the relevant range of these spreads.

The following argument establishes that leaper spreads should be less than 0.5%. Consider a 0.5% leaper with a 6% mortgage (i.e., a 6% mortgagor whose imputed coupon is 6.5%). We have shown that a financial engineer will refinance a 6.5% mortgage when market rates (of refinanceable mortgages) are in the 6.0% range. Accordingly a 0.5% leaper would refinance his 6% mortgage immediately at the time he receives the mortgage, a behavior which is clearly nonsensical. Prepayment of a 6% mortgage when rates are at 6% can be attributed only to turnover, not to refinancing. Therefore we will cap leaper spreads at 0.25%.

Laggard spreads on the other hand can be much wider, but should exceed 1.5% for only a very small percentage of the pool. A 1.5% laggard would not refinance his 6% mortgage until market rates have fallen below 4%. Given the readily available information about current mortgage rates (the so-called “media effect”), only a small number of people would not refinance a 6% mortgage when rates are at 4%.

While the behavior of mortgagors can vary widely, it is reasonable to assume that at the time of the inception of a new pool it is dominated by mortgagors in

the +25 to -25 bps range. But, as we discussed earlier, the distribution tends to become more and more laggardly as the pool ages.

7.2. The effect of laggards on value

Leaving all other assumptions (turnover = 75% PSA, refinancing cost = 1%, servicing cost = 0.125%, etc.) unchanged, Fig. 8 shows the fair coupon and the cost of the refinancing option for new 30-year mortgages over the relevant range of leapers

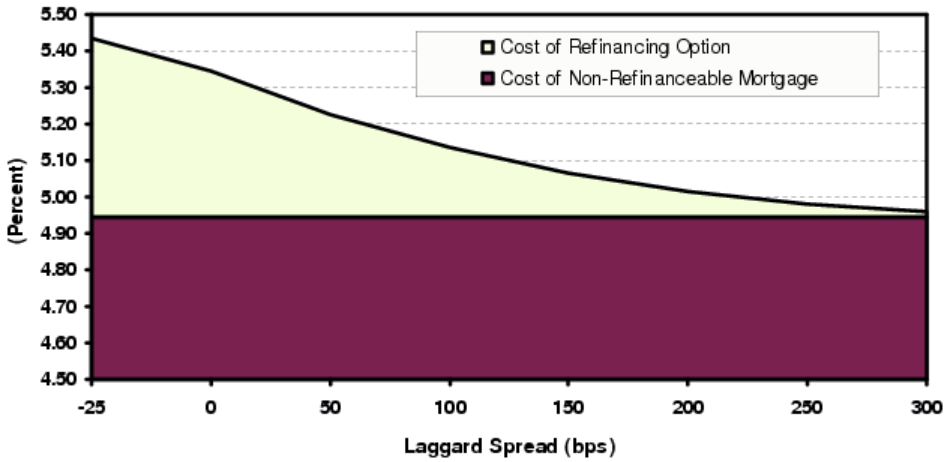


Fig. 8. Fair coupon (with 1/8 servicing fee) for 30-year refinanceable mortgage across behavior spectrum.

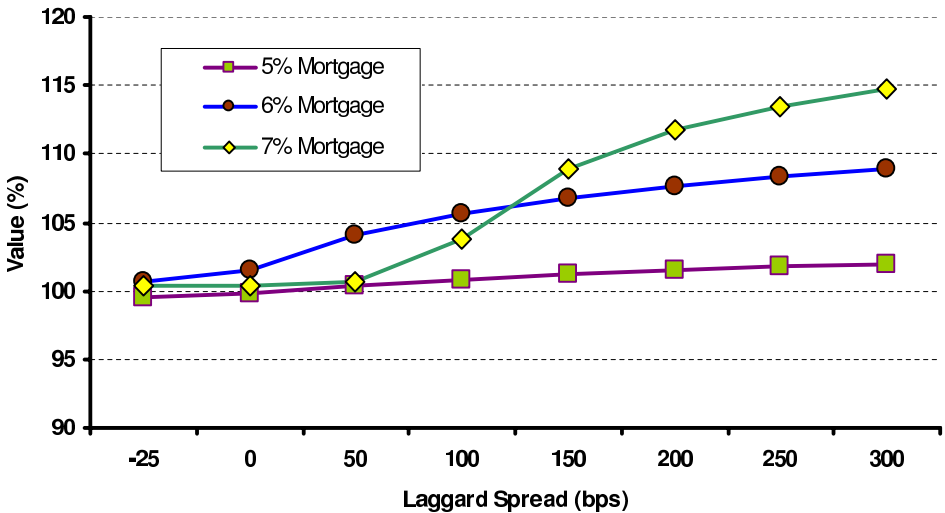


Fig. 9. Mortgage value depends on refinancing behavior.

and laggards. The cost of the option is obtained by subtracting the 4.95% optionless rate corresponding to 75% PSA (shown in Fig. 3). Evidently with the exception of extreme laggards, behavior has little impact on the cost. This confirms that the market rate of new mortgages can be explained by the assumption that financial engineers dominate the initial pool.

Figure 9 displays how refinancing behavior affects the values of 30-year 5%, 6%, and 7% unsecured mortgage pools and in the process also demonstrates how our approach captures the burnout phenomenon. For leapers or financial engineers, the value of a premium 7% mortgage is barely over par, but laggardly behavior greatly increases the value because the mortgages will remain outstanding longer. As the amount outstanding (i.e., the factor) declines, the mix automatically shifts towards laggards, increasing the dollar price of the amount that remains outstanding.

7.3. Laggard spread distribution

We have shown above how refinancing behavior, as depicted by laggard spread, affects the value of an unsecured mortgage pool. For a given distribution of laggard spreads, we can determine the value of the pool and that of the corresponding MBS. In this section we outline our approach for creating a laggard distribution.

Our recommendation is to infer the distribution from the market prices of actively traded securities. Given sufficient degrees of freedom, we could obtain a very good fit, but then the predictive power of the procedure would be low. Instead, we will confine ourselves to a very simple family of distributions, namely the negative exponential distribution anchored at 0 bps laggard spread.

In implementing the model, we represent the distributions by placing appropriate weights in evenly spaced buckets along the laggard axis. Figure 10 demonstrates the process. Starting with the financial engineers at the origin, the buckets

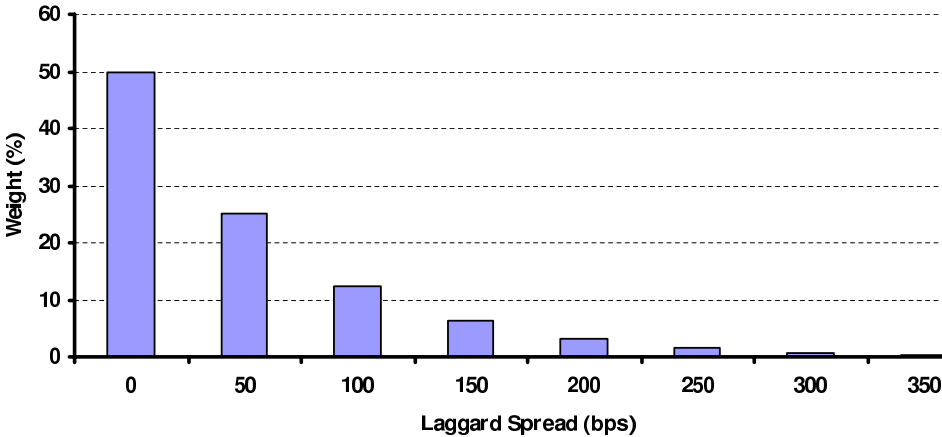


Fig. 10. Naïve laggard distribution.

are spaced at 50 bps intervals. We assume that the weight declines from bucket to bucket by a factor of 0.5. Accordingly the weight assigned to financial engineers will be 0.50 and the weight of 50 bps laggards will be 0.25, etc. We will use this distribution as a point of reference and refer to it as the “naïve” distribution.

We can modify the naïve distribution by either adjusting the spacing of the buckets (say to 40 bps) or the rate of decline (say to 0.6). While the two representations are theoretically equivalent, in practice one of them may turn out to be preferable to the other.

We close this section with an illustration of how seasoning impacts the value of an unsecured mortgage pool.

We consider pools of 30-year mortgages over a wide range of coupons, assuming that refinancing behavior follows the naïve distribution. Figure 11 shows the values of new pools (factor = 1.0), and of seasoned pools following major refinancing activity (factor = 0.5). Evidently the values are barely distinguishable for moderate coupons. In contrast, for high-coupon mortgages the value of a seasoned pool is much higher. In fact, as the coupon increases the value of a new pool actually declines, exhibiting negative duration. In case of a seasoned pool with a small factor, the same phenomenon would occur only at a much higher coupon level.

8. Valuation of MBS

8.1. How MBS differ from mortgage pools

So far we have focused on the valuation of unsecured mortgage pools; we now turn to MBS. All other factors being the same, there are two major reasons why an MBS is preferable to an uninsured mortgage pool. First, because it is guaranteed by the issuer (Fannie Mae or Freddie Mac) an MBS is more creditworthy. Second, because

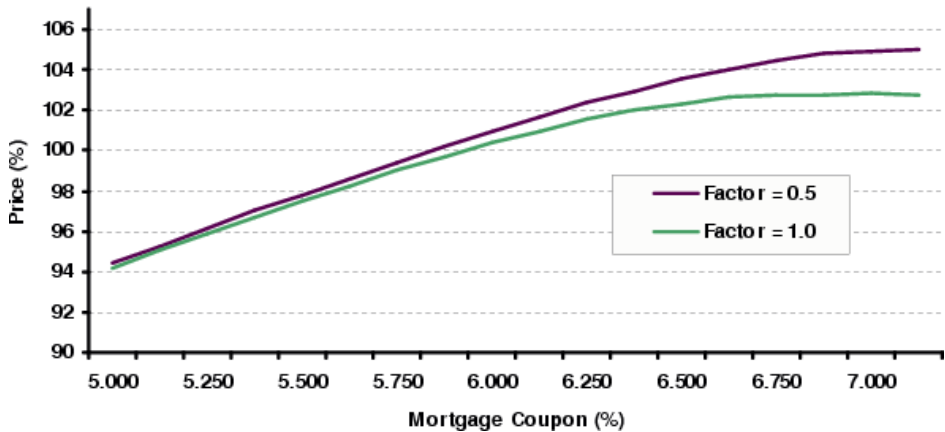


Fig. 11. How the factor affects the value of a pool of 30-year mortgages.

an MBS is a security, it is more liquid. For these reasons the cash flows of an MBS are discounted at a lower rate than those of an unsecured mortgage pool.

8.2. Valuation framework

We will represent the yield curves for an MBS and for a mortgage pool by respective OAS's relative to a benchmark swap curve. As long as the OAS's are fixed, the yield curves will be perfectly correlated. We have already established that the OAS of an MBS should be lower than that of its underlying pool. In the examples that follow we will assume that the OAS of every MBS is the same and demonstrate that even under this simplistic assumption we obtain robust and sensible results. Of course, in practice pool-specific OAS should be used.

What are reasonable values for the OAS's? The credit of a residential mortgage is comparable to a single-A rated corporate bond. Based on its duration, we estimate that currently the OAS of an unsecured mortgage pool should be roughly 70 to 90 bps to the swap curve. A more precise estimate from current mortgage rates can be obtained using the approach described in Sec. 6. In the valuation of MBS's, the fundamental role of the mortgage OAS is to project refinancing activity; the value of the mortgage pool is only of secondary interest. The higher this OAS the slower will be the rate of refinancing, and the greater will be the value of an MBS with an above-market coupon.

As discussed above, the OAS of an MBS should be significantly tighter than that of a mortgage pool. It should be comparable to the OAS of a debenture with similar duration and from the same issuer as the MBS. The current OAS of intermediate agency debentures is roughly 25 to 35 bps to the swap curve. This OAS has no effect on the cash flows; its sole function is discounting. The higher it is, the lower will be the value of the MBS.

Figure 12 shows the values of new MBS's. The coupon of the MBS is assumed to be 50 bps below the weighted average coupon ("WAC") of its mortgage pool.

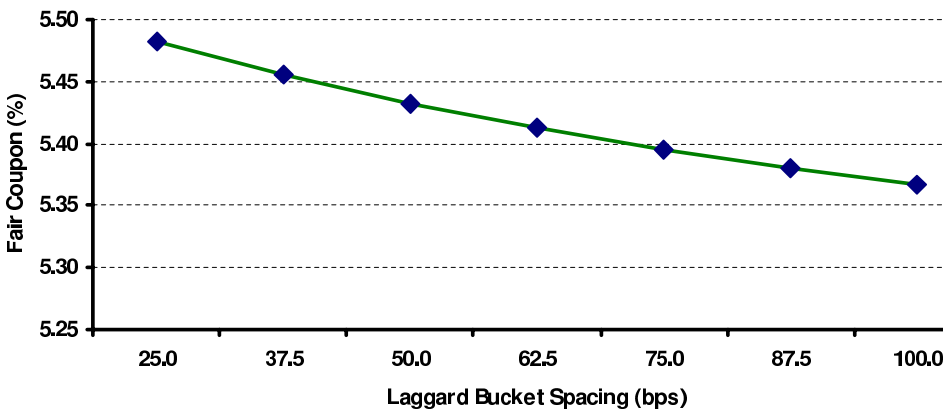


Fig. 12. Fair MBS coupon as laggard spacing changes (adjacent weights decline by 50%).

The mortgage OAS is 80 bps, the MBS OAS is 30 bps, and the naïve distribution is used to describe refinancing behavior (buckets placed 50 bps apart, weights of adjacent buckets decline by a factor of two).

8.3. Implied prepayment distribution

We close the paper with an application of our approach to real market prices. Table 1 shows the terms of 14 Fannie Mae MBS's along with their prices as of September 30, 2003 (courtesy of Countrywide Securities). Note the wide range of MBS coupons and pool factors. Table 2 shows the USD swap curve on the same day, and Table 3 displays the modeling assumptions.

Given the data in the tables, we determined the laggard distribution that provides the best fit to the given prices. In this application we created ten equally spaced buckets and assumed that the weights of adjacent buckets decline by a factor of two. We then varied the spacing of the buckets to determine the best fit.

Table 1. Fannie Mae prices of September 30, 2003.

MBS	WAC (%)	Original Amortization (mos)	Age (mos)	WAM (mos)	Factor	Price (%)
FNMA TBA 5.0	5.52	360	4	355	0.99	100.00
FNMA 2002 5.0	5.64	360	12	345	0.93	100.00
FNMA TBA 5.5	5.94	360	6	352	0.92	101.98
FNMA 2002 5.5	6.03	360	11	347	0.78	101.98
FNMA 2001 5.5	6.13	360	23	332	0.61	102.04
FNMA TBA 6.0	6.52	360	14	344	0.84	103.18
FNMA 2001 6.0	6.59	360	25	330	0.40	103.18
FNMA 1999 6.0	6.64	360	56	293	0.30	103.31
FNMA 1998 6.0	6.65	360	61	287	0.26	103.40
FNMA 2001 6.5	7.02	360	26	329	0.27	104.21
FNMA 1998 6.5	7.07	360	63	284	0.17	104.28
FNMA 1999 7.0	7.55	360	51	298	0.16	105.56
FNMA 1998 7.0	7.49	360	67	282	0.14	105.68
FNMA 2000 7.5	8.13	360	38	313	0.09	106.56

Table 2. Swap curve of September 30, 2003.

Term	1mo	3mo	6mo	1yr	2yr	3yr	5yr	10yr	30yr
Yield	1.160	1.160	1.180	1.290	1.886	2.498	3.374	4.495	5.303

Table 3. Assumptions.

Turnover rate	75% PSA
Refinancing cost	1% of mortgage
Mortgage OAS	80 bps
MBS OAS	30 bps
Short-rate volatility	16%
Mean reversion	0.0

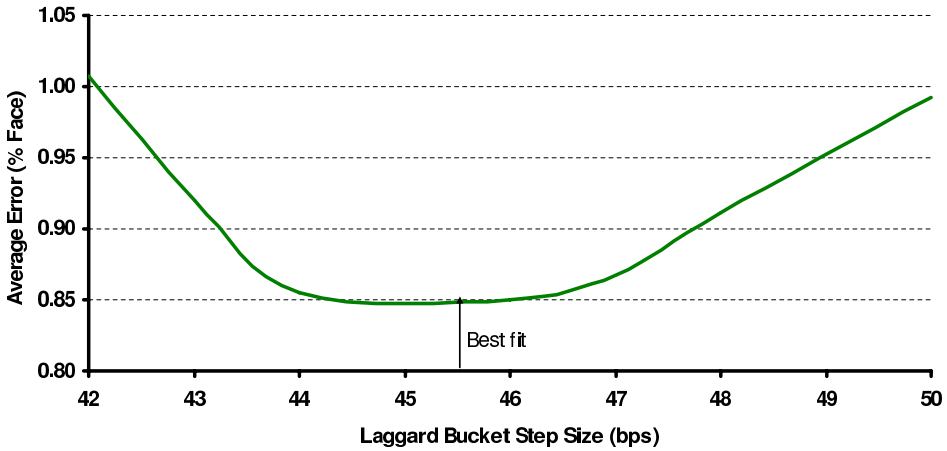


Fig. 13. Determining implied prepayment distribution (FNMA MBS prices of 9/30/03).

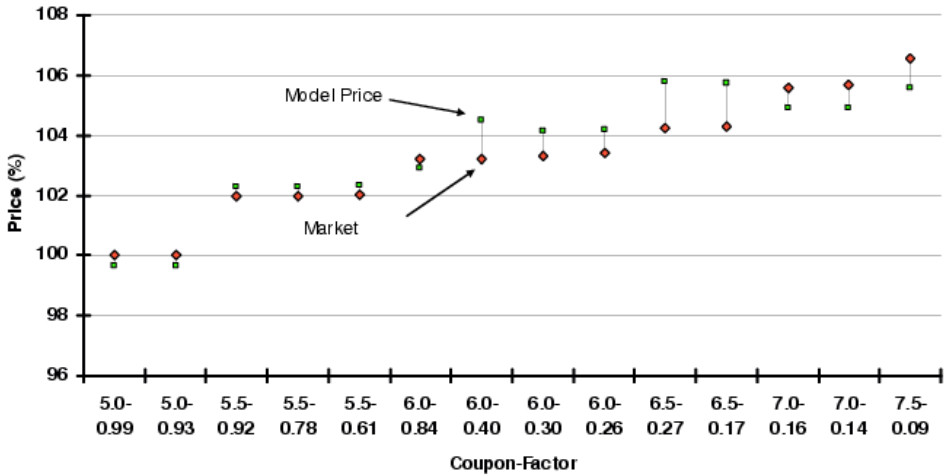


Fig. 14. FNMA MBS prices of 9/30/03.

As displayed in Fig. 13, the spacing that optimizes the fit occurs at 46 bps, and it results in an average error is 0.85%. Figure 14 shows the corresponding fitted values along with the actual prices of the MBS.

References

[1] F. Black, E. Derman and W. Toy, A one-factor model of interest rates and its application to treasury bond options, *Financial Analysts Journal* **46** (1990) 33–39.
 [2] W. M. Boyce and A. J. Kalotay, Optimum bond calling and refunding, *Interfaces* (November 1979) 36–49.

- [3] S. Chen, Implied prepayments: A new perspective on mortgage-backed securities, in *Fixed Income Solutions*, ed. T. Y. S. Ho (Irwin, 1996).
- [4] S. Chen, Understanding option-adjusted spreads: The implied prepayments hypothesis, *Journal of Portfolio Management* **22** (1996) 104–113.
- [5] O. Cheyette, Implied prepayments, *Journal of Portfolio Management* **23** (1996) 107–113.
- [6] Y. Deng, Mortgage termination: An empirical hazard model with stochastic term structure, *Journal of Real Estate Finance and Economics* **14** (1997) 309–331.
- [7] Y. Deng, J. M. Quigley and R. Van Order, Mortgage terminations, heterogeneity and the exercise of mortgage options, *Econometrica* **68** (2000) 275–308.
- [8] K. B. Dunn and J. J. McConnell, A comparison of alternative models for pricing GNMA mortgage-backed securities, *Journal of Finance* **36** (1981) 471–483.
- [9] K. B. Dunn and J. J. McConnell, Valuation of mortgage-backed securities, *Journal of Finance* **36** (1981) 599–617.
- [10] K. B. Dunn and C. S. Spatt, The effect of refinancing costs and market imperfections on the optimal call strategy and the pricing of debt contracts, working paper, Carnegie-Mellon University (1986).
- [11] A. Davidson and M. Hershovitz, The refinancing threshold pricing model: An economic approach to valuing MBS, Merrill Lynch Mortgage-Backed Research (November 1987).
- [12] C. D. Howard and A. J. Kalotay, Embedded call options and refunding efficiency, in *Advances in Futures and Options Research*, Vol. 3 (JAI Press, 1988) 97–117.
- [13] J. D. Finnerty and D. R. Emery, *Debt Management* (Harvard Business School Press, 2001).
- [14] J. D. Finnerty, A. J. Kalotay and F. X. Farrell Jr., *Evaluating Bond Refunding Opportunities*, The Institutional Investor Series in Finance (Ballinger, 1988).
- [15] L. Hayre (ed.), *Salomon Smith Barney Guide to Mortgage-Backed and Asset-Backed Securities* (John Wiley, 2001).
- [16] J. Hilliard, J. B. Kau and C. Slawson, Valuing prepayment and default in a fixed-rate mortgage: A bivariate binomial options pricing technique, *Journal of Real Estate Economics* **26** (1998) 431–468.
- [17] J. Hu, An alternative prepayment projection based on housing activity, in *The Handbook of Mortgage Backed-Securities*, ed. F. J. Fabozzi (Probus Publishing, 1988).
- [18] E. Johnston and L. Van Drunen, Pricing mortgage pools with heterogeneous mortgagors: Empirical evidence, working paper, University of Utah (1988).
- [19] A. J. Kalotay, G. O. Williams and F. J. Fabozzi, A model for valuing bonds and embedded options, *Financial Analysts Journal* **49** (1993) 35–46.
- [20] J. B. Kau and V. C. Slawson Jr., Frictions, heterogeneity, and optimality in mortgage modeling, *The Journal of Real Estate Finance and Economics* **24** (2002) 239–260.
- [21] R. Klotz and A. Shapiro, Dealing with streamlined refinancings: New implied prepayment model, Merrill Lynch Mortgage Product Analysis (October 1993).
- [22] A. Levin, Active-passive decomposition in burnout modeling, *Journal of Fixed Income* **10** (2001) 27–40.
- [23] R. Roll and S. F. Richard, Modeling prepayments on fixed rate mortgage-backed securities, *Journal of Portfolio Management* **15** (1989) 73–82.
- [24] K. Sandmann and D. Sondermann, A note on the stability of lognormal interest rate models and the pricing of eurodollar futures, *Mathematical Finance* **7** (1997) 119–128.
- [25] R. Stanton, Rational prepayment and the valuation of mortgage-backed securities, *Review of Financial Studies* **8** (1995) 677–708.

- [26] R. Stanton and N. Wallace, Mortgage choice: What's the point?, *Real Estate Economics* **26** (1998) 173–205.
- [27] G. C. Timmis, Valuation of GNMA mortgage-backed securities with transaction costs, heterogeneous households and endogenously generated prepayment rates, working paper, Carnegie-Mellon University (1985).