OR/MS Today - June 2007

Financial O.R.

A Pointer on Points

Given the array of residential mortgage products, should a homebuyer pay up-front points in order to lower the annual percentage rate? Introducing the notion of option adjusted APR.

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Residential mortgages today are offered to homeowners in a large variety of structures. A borrower can choose among different maturities, with 15- and 30-year mortgages being the most popular. The interest rate can be fixed, adjustable or a combination of the two as in the hybrid 30-year adjustable rate products that have a fixed rate for the first five years. A third variable is the amortization of principal, which can be conventional or involve a balloon payment. An extreme example of the latter is the interest only loan (for the first five or 10 years).

After choosing from among the plethora of loan products available, the borrower faces a further choice. Should he pay some up-front amount to lower the nominal interest rate? This amount is usually expressed as a percentage of the total borrowing principal, and is referred to as points. For example, a 5.5 percent, 30-year mortgage with an up-front 0.75 point simply means that by paying 0.75 percent of the principal up front, the 30-year fixed rate would be reduced to 5.5 percent (vs. a standard 30-year fixed rate, at the time 6.0 percent). Should points be paid? We examine this problem and offer a solution.

First, we'll show that annual percentage rate (APR), the standardized measure mandated by "truth in lending" disclosure statutes, does not tell the whole truth, because it does not take into consideration the option to refinance if interest rates decline. Second, we'll explain how to deal with interest rate uncertainty when the borrowing horizon coincides with the nominal maturity of the loan. Later, we'll consider the significant effects of the borrowing horizon and personal income taxes. The horizon can turn out to be shorter than the nominal maturity of the mortgage, either because the homeowner never intended to stay for the full term or because of the unpredictable factors unrelated to interest rates.

In all the examples below, we assume that the borrower will consider only 30-year fixed rate loans. It will be clear, however, the proposed analytical framework is applicable to any standard mortgage, fixed, floating or hybrid of any nominal maturity.

Does It Pay to Pay Points?

In addition to quoting a rate for a mortgage with no points, most lenders also offer lower rates in exchange for up-front points. Table 1 displays the rates posted by a major bank for 30-year mortgages during the summer of 2005.

Table 1:

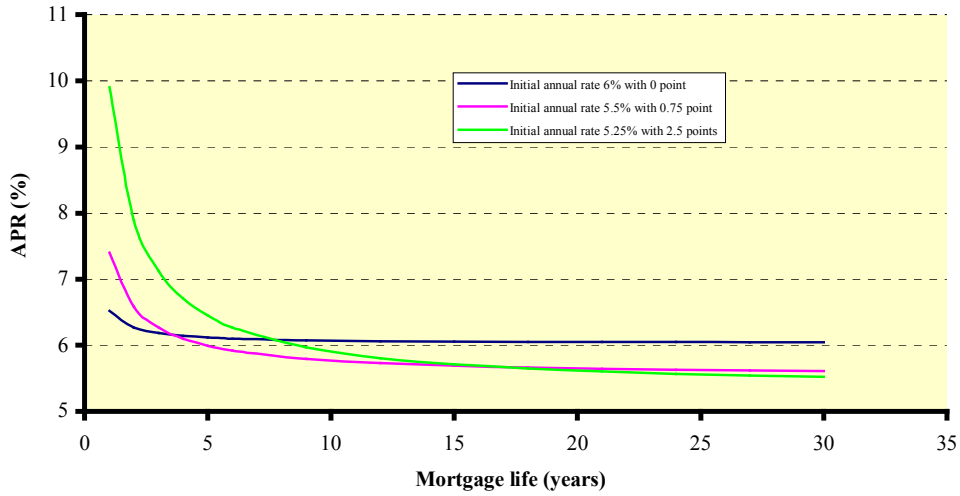
30-year fixed rate mortgage alternatives.		
Points	**Annual Rate (%)**	**APR (%)**
0	6.000	6.047
0.75	5.500	5.614
2.50	5.250	5.524

The indicated APR is the internal rate of return of the cash flows assuming that the mortgage remains outstanding for its full scheduled life and that the closing costs is 0.5 percent of principal. In other words, assuming that the mortgage remains outstanding for 30 years, APR is the discount rate that equates the sum of all the discounted cash flows to the proceeds. In general, the APR takes into account any points and origination fees and private mortgage insurance associated with the loan agreement.

However, APR does not tell the whole story, because it is based on the assumption that the loan remains outstanding for its full term. In fact, the actual life of the loan can turn out to be much shorter. For starters, homeowners can move and repay the loan by selling their home. Another possibility — the key consideration in this paper — is that mortgage rates may decline and afford the homeowner an opportunity to prepay and refinance at a lower cost.

In order to gain an appreciation of how APR depends on the actual life, let's take a look at Figure 1. Clearly, the greater the up-front points, the more sensitive is the APR to the actual life. We observe that the 6 percent par mortgage and the 5.5 percent mortgage with a 0.75 percent discount intersect shortly after year three, so the 0.75 percent discount mortgage is preferable only if the mortgage remains outstanding for at least three years. Similarly, the par and the 5.25 percent mortgage with a 2.5 percent discount intersect approximately at year nine, and the 0.75 percent discount mortgage and 2.5 percent discount mortgage cross paths at around year 18. In summary, if the actual life turns out to be less than three years, the par mortgage is preferable; if the life is 18 years or longer, the 5.25 percent discount mortgage is the clear winner; and between three years and 18 years, the choice should be the 5.5 percent discount mortgage.

Figure 1: APR Depends on Life of the Mortgage



So why would one choose the par mortgage over one with a discount? Obviously not because one's borrowing horizon is less than three years — for that, there are better alternatives than 30-year fixed rate mortgages. But if one expects rates to substantially decline within the next three years, the par mortgage is the most effective funding vehicle among the three alternatives. If rates decline within the first three years, the *blended cost* of the par mortgage and its replacement will be lower than the blended cost of a discount mortgage and its replacement. Therefore, the strategy of choosing the seemingly costlier par mortgage would lead to cheaper overall borrowing.

The problem is that at the outset the future course of mortgage rates is unknown. Nevertheless, we can employ standard bond valuation tools [3, 4, 5] to identify the strategy with the lowest expected cost, taking into account future refinancing opportunities. Our basic approach is to model interest rates as a stochastic process and assume that the homeowner will refinance whenever it is optimal. (For details in valuing bonds with options and refunding efficiency, see [3, 6. 7]).

Dealing with Interest Rate Uncertainty

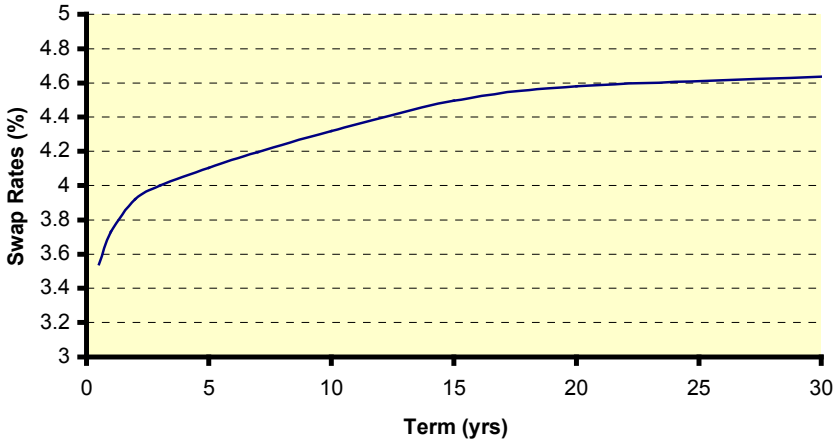
For now, assume that the homeowner's borrowing horizon coincides with the term of the mortgage. Because conventional mortgages can be prepaid at any time without penalty, if interest rates sufficiently decline the homeowner will prepay and refinance at the prevailing lower rate. Of course, the decision should take into account any transaction costs. Obviously in choosing among the loans, the homeowner should consider the possibility of refinancing down the road. (For a detailed discussion of the refinancing methodology, see [8]).

The decision is based on comparing the savings (in terms of net present value, after transaction costs) to the forfeited net option value. Their ratio, the so-called refinancing efficiency, cannot exceed 100 percent. In this paper we assume that the loan is refinanced whenever it is optimal to do so, i.e., when the efficiency reaches 100 percent.

We represent a mortgage as a callable amortizing bond that pays interest monthly. The call premium in excess of par is determined by the cost of refinancing; in the examples below we assume that it is 1 percent of the remaining principal.

We model the evolution of interest rates by the industry-standard Black-Karasinski process. (For a description of the model, see [1, 2]). The results below are based on a 16 percent short-rate volatility, which is in line with the implied volatilities of agency debentures of similar duration. For a benchmark yield curve we use the current swap curve (see Figure 2). We should point out, however, that the resulting ranking of the loans is rather insensitive to the choice of the benchmark curve.

Figure 2: US Swap Curve on June 8, 2005



Given the above assumptions, we can employ standard bond valuation technology to determine the expected cost of each strategy, i.e., the expected cost of each loan combined with its replacement in the event of future refinancing. As a starting point, we need a yield curve of option-less mortgage rates. Because such yield curve is not directly observable, we "calibrate" the benchmark swap curve to each mortgage alternative. Specifically, we find the option adjusted spread (OAS) that explains the mortgage proceeds. Comparison of the results enables us to identify the loan with the lowest OAS, which is the most desirable from the perspective of a borrower.

Although for a bond professional, identifying the loan with the lowest OAS would be the end of the exercise (except for confirming that the option-adjusted durations of the alternatives are in the same ballpark), in the case of fixed-rate mortgages we can go one step further. Since the OAS concept is very alien to homeowners, we will express the costs of the mortgages using the APR terminology that retail borrowers are already familiar with, but refine the definition to reflect the option to refinance.

Specifically, we will introduce the notion of *option adjusted APR* and use it to measure the cost of each strategy. It is the APR based on the *sum* of the proceeds and the option value. Option-adjusted APR is analogous to the so-called option adjusted yield to maturity in the bond world. Recall that the traditional APR is based solely on the proceeds; it does not consider the option value.

In order to make the comparison fair, we need to apply the same OAS to each mortgage. In the examples that follow, we use the OAS of the par mortgage. (Since this is a *relative* analysis, any one of the computed OASs may be used without altering the ranking.) Based on this OAS, we compute the option value of each mortgage and add it to the proceeds. In essence, this summation captures the fact that in addition to the loan, the borrower also receives an option to refinance the loan should the opportunity arise. The final step is to compute the APR of the "full value" of the mortgage. Ceteris paribus, the optimal strategy for the homeowner is to choose the loan with the lowest option adjusted APR.

As mentioned above, the origination cost of the mortgages is assumed to be 0.5 percent of the principal amount. We incorporate this fee in the analysis that follows. Based on the pricing of the par mortgage, we determine that its OAS is 76.6 basis points. Table 2 displays the traditional APRs, option values and option adjusted APRs for the three mortgages. Note that when we ignore the option, according to the traditional APR, the 5.25 percent mortgage (the one that pays the most points) is the most attractive. But when we consider the possibility of refinancing, the optimal strategy is the 5.5 percent mortgage. The result agrees with the ranking based on OAS, as expected.

Table 2:

Results for selected 30-year fixed-rate mortgages (computed with 76.6 bps OAS and 16 percent volatility).

Points	Annual Rate (%)	Traditional APR (%)	Option Value (% of par)	Option Adjusted APR (%)
0	6.0	6.047	9.113	5.247
0.75	5.5	5.614	5.829	**5.100**
2.5	5.25	**5.524**	4.558	5.113

If we repeat the analysis with the volatility reduced from 16 percent to 12 percent, the 5.25 percent mortgage (the one with highest points) is the most cost efficient even after the option adjustment, as shown in Table 3. This is because the option values decrease as volatility is reduced.

Table 3:

Results for three selected mortgage products (computed with 107.1 bps OAS and 12 percent volatility).

Points	Annual Rate (%)	Traditional APR (%)	Option Value (% of par)	Option Adjusted APR (%)
0	6.0	6.047	5.632	5.540
0.75	5.5	5.614	3.022	5.342
2.5	5.25	**5.524**	2.167	**5.325**

As to what is the right volatility to use, we need to look at the implied volatilities of agency debentures of similar duration. As we mentioned earlier, 16 percent is a reasonable number to use under the current market condition.

Personal Income Taxes and Borrowing Horizon

The choice of the mortgage should be made on an after-tax basis, as taxes affect both cash flows and option values. The IRS allows deduction of interest payments and discount points (on the original mortgage); however closing costs are not deductible.

We take personal incoming taxes into consideration by computing OAS and option value in an after-tax framework. The results are shown in Table 4. On an after-tax basis, the OAS for the par mortgage is 78.9 bps. Because of that, the tax benefit associated with the 2.5 discount points — the deeply discounted mortgage — is the best choice.

Table 4:

After-tax results for selected 30-year mortgages (computed with 38 percent tax rate, 78.9 bps OAS and 16 percent volatility).

Annual Rate and discount points (%)	Traditional APR (%)	After Tax Option Value (% par)	After Tax Proceeds +Option Value (% par)	Taxable Equivalent Option Adjusted APR
6.00 (0.00)	6.047	6.999	106.499	5.423
5.50 (0.75)	5.614	4.744	103.779	5.168
5.25 (2.50)	**5.524**	3.841	101.791	**5.092**

The analysis so far assumed that the borrower would stay in the home until the nominal maturity of the mortgage. The actual stay, however, may turn out to be shorter, due to planned or random developments. For example, consider a "planned" case: the homeowner intends to move in 18 years, when his now newborn daughter goes to college. In order to keep his monthly payment low and predictable, he obtains a fixed rate 30-year mortgage, even though his time horizon is shorter.

Which mortgage should he choose now? To answer this question, we calculate the option-adjusted APRs assuming an 18-year horizon. The results are shown in Table 5.

Table 5:

Results for selected 30-year mortgages with 18-year horizon (computed with 38 percent tax rate, 94.0 bps OAS and 16 percent volatility).

Annual Rate and discount points (%)	Traditional APR (%)	After Tax Option Value (% par)	After Tax Proceeds +Option Value (% par)	Taxable Equivalent Option Adjusted APR
6.00 (0.00)	6.047	5.110	104.610	5.536
5.50 (0.75)	5.614	3.291	102.326	5.268
5.25 (2.50)	**5.524**	2.576	100.526	**5.197**

There are, of course, many other factors that determine the actual borrowing horizon, as opposed to the one planned at the outset. These include job changes, marriage, divorce and children. How do we deal with such unpredictable factors? A simple yet reasonable approach is to assume that the homeowner is subject to the same demographical considerations as the population on the average.

A widely used benchmark for valuing the turnover is the Public Security Association (PSA) standard prepayment benchmark. It is expressed as a monthly series of annual constant prepayment rates (CPR). The basic PSA model assumes that the prepayment will be low when the mortgage just originated, then it will grow linearly, and after a period of time it will stay stable. For a 30-year mortgage, 100 percent PSA assumes that prepayment rate starts at 0 percent CPR at age 0 and increases by 0.2 percent CPR per month until month 30, after which the speed is a constant 6 percent annual CPR. A rate of 50 percent PSA means that CPR in any month is half of that implied by 100 percent PSA; a rate of 200 percent PSA means that CPR in any month is twice of that implied by 100 percent PSA, and so on.

We can incorporate this PSA speed into our analysis to calculate a probabilistic option-adjusted APR for each mortgage. However, we will leave the details to another time.

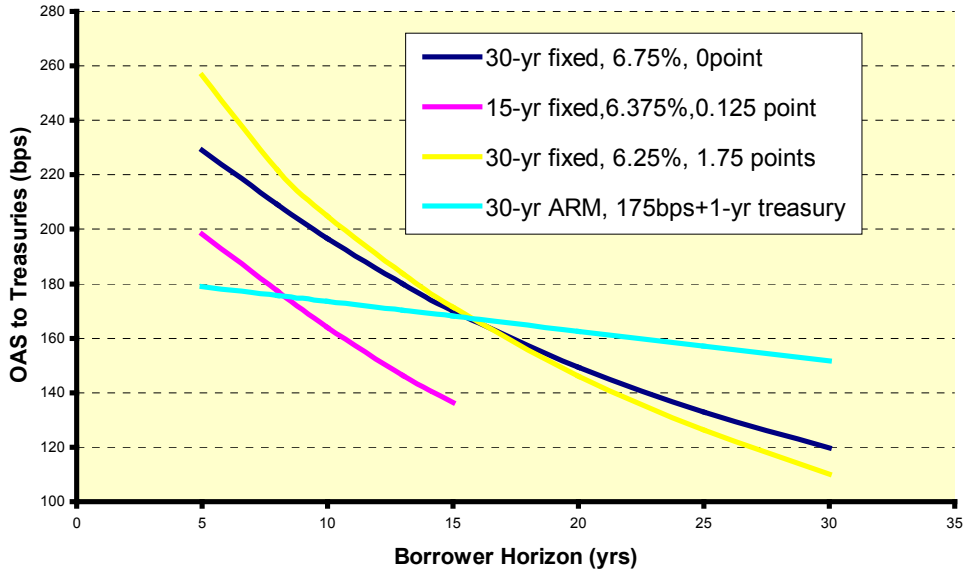
Conclusion

Homeowners today are offered a wide menu of mortgage alternatives. In particular, they can reduce the nominal interest rate, as measured by the APR, by paying up-front points. The shortcoming of the APR is that it fails to take into account the possibility of refinancing in the event mortgage rates decline.

Identifying the mortgage with the lowest expected cost is a formidable task that confronts millions of borrowers. The fundamental problem is to incorporate the value of the refinancing option into the decision-making process. We have shown how standard bond valuation tools can be employed to provide a solution, which is reported as an *option-adjusted APR*. The approach can incorporate borrower-specific parameters (borrowing horizon and personal income tax rate) to derive the *taxable equivalent option adjusted APR* over the specified horizon.

Although the examples were confined to long-term fixed rate mortgages, the methodology is also applicable to adjustable rate structures. In Figure 3, we show the OAS to treasuries for four different mortgage products, which include different maturity fixed rates mortgages and also adjustable rate mortgage. The result is based on market information of Nov. 2, 2005. In summary, we believe the OAS technology brings a badly needed professional discipline to the mortgage selection process.

Figure 3: Expected Cost Depends on the Borrowing Horizon



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