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Refunding efficiency: a generalized approach

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Refunding efficiency, a measure of the optimality of a refunding decision, is widely used in the call exercise decision for agency, corporate and municipal bonds. The original definition of efficiency assumes that the refunding bond is optionless. However, in practice, the refunding bond is often callable. We show that the commonly used method of incorporating the value of the refunding bond's call option into the efficiency calculation can lead to paradoxical results, and then suggest a new definition of refunding efficiency that overcomes this problem.

I. Introduction

Call provisions are a common feature of bonds issued in the United States. Most of the roughly 15 000 agency bonds outstanding are callable, as are most municipal bonds whose maturity exceeds 10 years. Until recently the majority of long-term bonds issued by electric and telephone utilities also included call provisions. The total volume of callable bonds outstanding is several trillion dollars.

In recent years as interest rates declined, the call/refunding volume in the US government agency and municipal sectors has been running at a pace of hundreds of billions of dollars annually. This volume includes the so-called advance refunding (defeasance) of municipal bonds, which is effectively a call exercise prior to the initial call date (Kalotay and May, 1998). The associated transaction cost has been of the order of billions of dollars per year. Clearly, bond refunding is big business, and issuers of callable bonds are inundated by investment banking proposals recommending related transactions. Many of these proposals routinely provide the so-called *refunding efficiency* of the transaction.

According to standard option pricing theory, a bond should not be called until the savings achieved by refunding equals the value of the call option. However, risk-averse issuers may not want to wait until the absolute optimal moment and will sometimes call the bond prematurely. A measure of how close a premature call is to being optimal is therefore useful.

Boyce and Kalotay (1979) established a rigorous approach to this problem. They introduced the notion of refunding efficiency, which is the ratio of savings from (expressed throughout this article in present value terms) to the value of the call option forfeited through exercise. Risk-neutral treasurers will 'pull the trigger' only when efficiency reaches 100%.

In the original paper, Boyce and Kalotay suggested 85% as the minimum acceptable efficiency. With the advent of the interest rate derivatives market allowing for synthetic 'monetization' of the call option, that threshold should obviously be raised considerably. These days, in fact, sophisticated issuers are reluctant to call below 100% efficiency i.e., when the embedded option is worth more alive than dead.

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Because refunding efficiency today is widely used by decision makers and included in investment banking proposals, it is critical to calculate it correctly. Unfortunately, as we shall show, the current industry practice can overstate the refunding efficiency when the replacement bond is itself callable. This results in premature call exercise and a waste of optionality.

In the next section, we first demonstrate how refunding efficiency should be calculated and used when the new bond is non-callable. We then describe current industry practice for calculating refunding efficiency when the refunding bond is callable. Although at first blush this method seems reasonable, we show that it can result in a misleading recommendation. In Section III, we propose a new definition of refunding efficiency. While being consistent with the original definition for cases where the refunding bond is optionless, it overcomes the problem of possibly overstating efficiency when the refunding bond is callable.

II. Refunding Efficiency

Boyce and Kalotay (1979) originally defined the refunding efficiency of a calling bond assuming that the bond would be refunded with a maturity-matched non-callable bond.[1] They defined refunding efficiency as

$$\text{Efficiency}_{\text{Original}} = \frac{\text{Cash flow savings}}{\text{Forfeited option value}}, \quad (1)$$

where 'cash flow savings' is the present value of the differences in the bond cash flows plus transaction cost and 'forfeited option value' is the value of the call option embedded in the outstanding bond. Since the former – the intrinsic value of the option – cannot exceed the latter, the efficiency cannot exceed 100% and is equal to 100% when and only when it is optimal to refund. Boyce and Kalotay's notion of refunding efficiency has become the standard for deciding whether to refund a bond or not (see, for example, Finnerty and Emery, 2001).

The savings in this formula can be determined by discounting the differential cash flows. Indeed, even the need to discount can be avoided if the cash flows of the (non-callable) refunding issue are identical to the remaining cash flows of the outstanding issue. In this case, the savings are the proceeds of the new issue less the cost of calling (call price plus transaction costs).

Table 1. Non-callable par yield curve for agency

Term	2-year	5-year	7-year	10-year	15-year	30-year
Rate (%)	3.20	3.97	4.32	4.67	5.06	5.33

Suppose that an agency is considering refunding a 4.25% bond with 5 years left to maturity, callable at any time at 100. Suppose further that the agency could issue 5-year non-callable bonds at 3.97%. The transaction cost associated with issuing the new bond would amount to 0.375% of the principal. Calculation of the refunding efficiency requires an interest rate model calibrated to the issuer's yield curve and set to the appropriate volatility.[2]

Table 1 displays the agency's non-call par yield curve (for this example yields beyond 5 years are irrelevant). While the details of how to choose the appropriate volatility is beyond the scope of this article, we note that for agency bonds the current practice is to infer the volatility from the swaptions market. In the subsequent examples we assume that interest rates follow a lognormal random walk with a short-term volatility of 16%.

Based on the above information, we determine that the present value of the remaining cash flows out to maturity is 101.271% and the option value is 1.164% of the outstanding principal. Thus savings would amount to $101.271\% - (100\% + 0.375\%)$, or 0.896%. Therefore the refunding efficiency is $0.896\%/1.164\% = 76.98\%$, which is significantly below the optimal 100%. We conclude that it would be premature to call these bonds.

How high would the coupon of the outstanding bond have to be to justify calling? We can repeat the calculations over a range of coupon levels, as shown in Fig. 1. Note that (i) both the savings and option value increase with the coupon, (ii) the savings cannot exceed the option value, (iii) initially the savings increase faster than does the option value and (iv) the savings and the option value are equal (at 1.94% of outstanding principal) when the coupon is 4.48%. The resulting refunding efficiencies are shown in Fig. 2, from which we conclude that the agency should immediately refund any 5-year bond with a coupon above 4.48%, and leave those with a lower coupon outstanding.

[1] Howard and Kalotay (1988) employed the same approach in a more accessible article.
[2] Throughout this article all option values are computed using a lognormal short rate model (i.e. the Black *et al.* (1990) or Black *et al.* (1991) model with zero mean reversion) with constant volatility and calibrated to the appropriate yield curve.

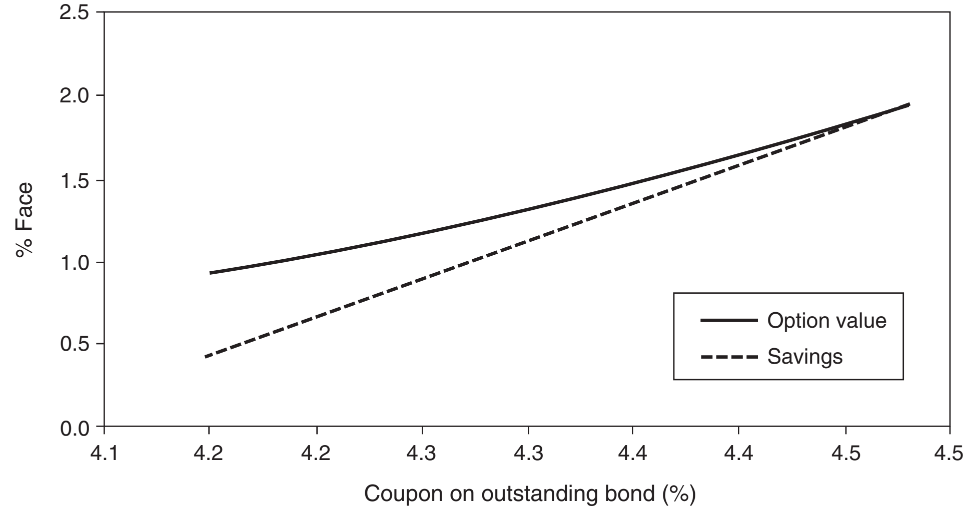

Fig. 1. Five-year bonds refunded with 3.97% maturity-matched bullet

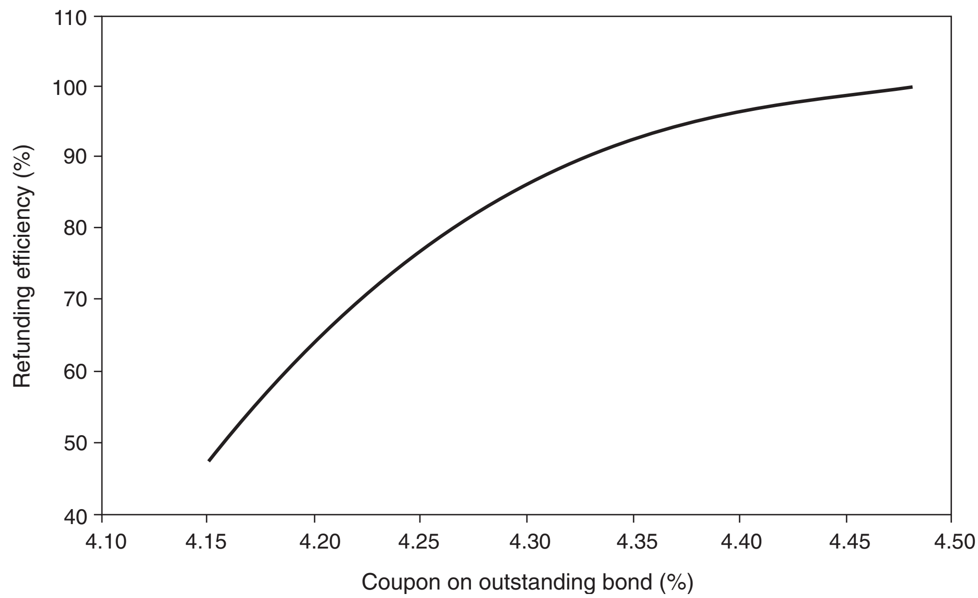

Fig. 2. Refunding efficiency of 5-year bonds refunded with a 3.97% maturity-matched bullet

In practice, the new issue is different from the textbook case; in particular it may also be callable. For example, the housing agencies (Freddie Mac, Fannie Mae and the Federal Home Loan Banks) routinely refund with bonds that are callable at par, often within 3 months from the date of issue.

Quantifying savings is more complicated when the new bond is callable because the possibility of future refundings, including the transaction costs, must also be accounted for. Thus the value of the embedded call option of the refunding issue needs to be incorporated into the efficiency formula. The question is how.

The current industry practice is to define efficiency when the new bond is callable as

$$\text{Efficiency}_{\text{Original}} = \frac{\text{Change in Cash Flow Value} + \text{Option Value}_{\text{New}}}{\text{Option Value}_{\text{Old}}} \quad (2)$$

where 'change in cash flow value' is the difference between the old and new bond cash flows less transaction costs. This definition still has the desirable property that for any plausible refunding it never exceeds 100%, and is equal to 100% if and only if it is optimal to refund the bond.

Continuing with the example from earlier, let us demonstrate the use of this formula. Suppose that the refunding bond matures in 5 years, is callable at 100 any time after the end of the first year, and has a 4.142% coupon. The PV of the cash flows of this 4.142% bond is 100.781% and its option value is 0.781%, resulting in a fair value of 100.781% − 0.781%, or 100%.[3] Thus the efficiency is (0.115% + 0.781%)/1.164% = 76.98%, the same as in the case when the refunding bond is non-callable. This agreement may provide a false sense of comfort.

To see that this definition of efficiency is in fact unsatisfactory, consider the scenario of refunding with a new bond that is identical to the outstanding one in all aspects, including the call option. This transaction would be clearly undesirable, because the prospective cash flows remain the same but transaction costs have been incurred. Yet the definition assigns a positive efficiency to this transaction. If the transaction costs are a small fraction of the option value, the efficiency can even be close to 100%, which is clearly a misleading result.

III. Correcting the Refunding Efficiency Formula

The correct formula is obtained by considering the effect of the transaction at the margin and defining

$$\text{Efficiency}_{\text{New}} = \frac{\text{Change in cash flow value}}{\text{Change in option value}}. \quad (3)$$

We now prove that for any plausible refunding transaction

$$\text{Efficiency}_{\text{New}} \leq \text{Efficiency}_{\text{Original}} \leq 100\% \quad (4)$$

Proof: A callable bond decomposes into a bullet bond, consisting of the scheduled cash flows out to maturity, and a call option; therefore,

$$\text{Bond value} = \text{Bullet value} - \text{Option value}.$$

We will also assume next that the call price of the outstanding bond is equal to the value of the refunding bond and that the cost of refunding is an upfront cash flow.

First, observe that refunding a bond is worth consideration only if the quantity

$$\begin{aligned}\text{Change in cash flow value} &= \text{Bullet value}_{\text{Old}} - \text{Bullet value}_{\text{New}} \\ &\quad - \text{Refunding cost}\end{aligned}$$

is positive, to guarantee cash flow savings even if the new bond is never refunded. On the other hand,

$$\begin{aligned}\text{Option value}_{\text{Old}} &\geq \text{Bullet}_{\text{Old}} - \text{Call price} - \text{Refunding cost} \\ &= \text{Bullet}_{\text{Old}} - \text{Bullet}_{\text{New}} + \text{Option value}_{\text{New}} \\ &\quad - \text{Refunding cost}.\end{aligned}$$

Therefore,

$$\begin{aligned}\text{Change in option value} &\geq \text{Bullet}_{\text{Old}} - \text{Bullet}_{\text{New}} - \text{Refunding cost} \\ &= \text{Change in cash flow value}.\end{aligned}$$

This shows that the efficiency defined by Equation 3 never exceeds 100%.

If we subtract Equation 3 from Equation 2, a straightforward calculation shows that

$$\begin{aligned}\text{Efficiency}_{\text{Original}} - \text{Efficiency}_{\text{New}} \\ = \frac{\text{Option value}_{\text{New}}}{\text{Change in option value}}(1 - \text{Efficiency}_{\text{Original}}),\end{aligned}$$

which proves inequality (4).

The new definition has several other desirable properties. Refunding with an identical callable bond has infinitely negative efficiency. This accords with the intuition that it is a losing transaction, unlike the non-sensical result of the formula currently in use. If the new issue is non-callable, the definition reduces to the original one. Another attractive feature is that it extends naturally to portfolios, defining the efficiency of a portfolio restructuring

Let us apply the new definition to the example considered at the end of Section II. The cash flow savings is 101.271% − (100.781% + 0.375%) = 0.115%, while the change in option value is 1.164% − 0.781% = 0.383%. Therefore the efficiency is 30.03%, which is much lower than the 76.98% efficiency according to the formula currently used in industry.

Figures 3 and 4 are analogous to Figs 1 and 2. Fig. 3 shows the savings and change in option value, and Fig. 4 shows the corresponding refunding efficiency.

[3] Here, we are assuming that the refunding bond is priced fairly at par. It is beyond the scope of this article to discuss what to do if it is not, as is commonly the case for retail notes.

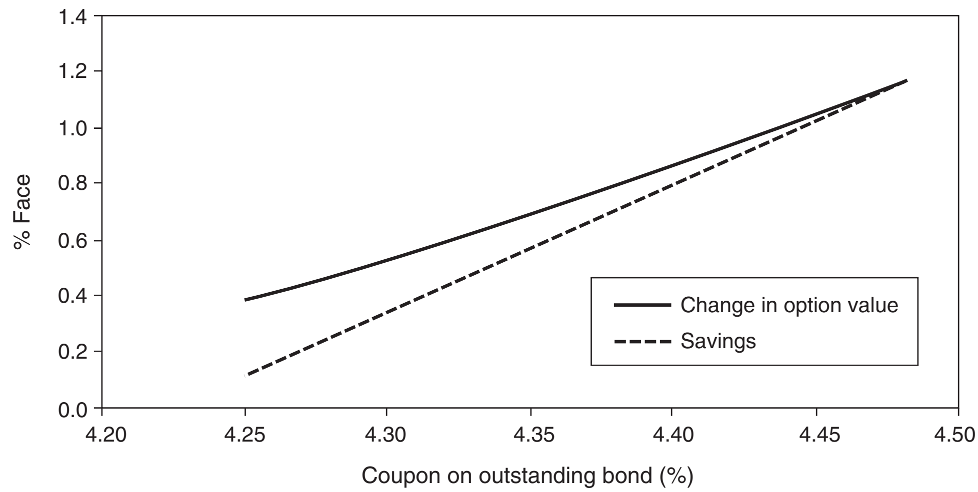

Fig. 3. Five-year bonds refunded with maturity-matched callable 4.142% bond

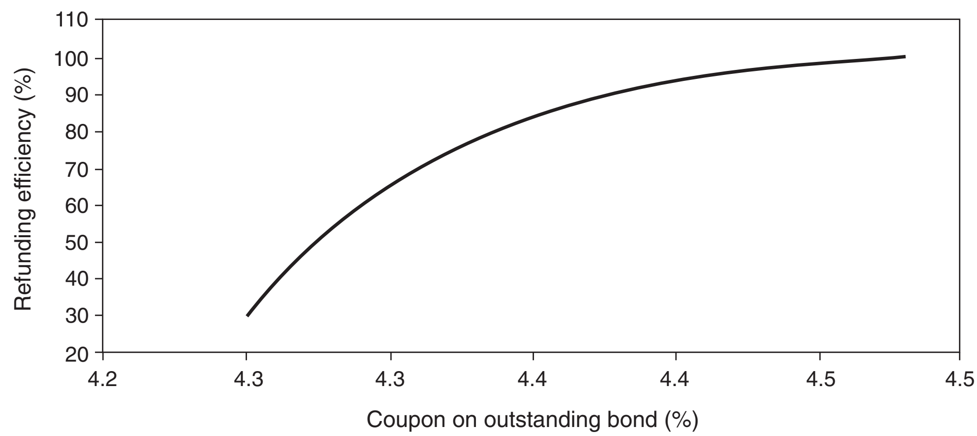

Fig. 4. Refunding efficiency of 5-year bonds refunded with maturity-matched callable 4.142% bond

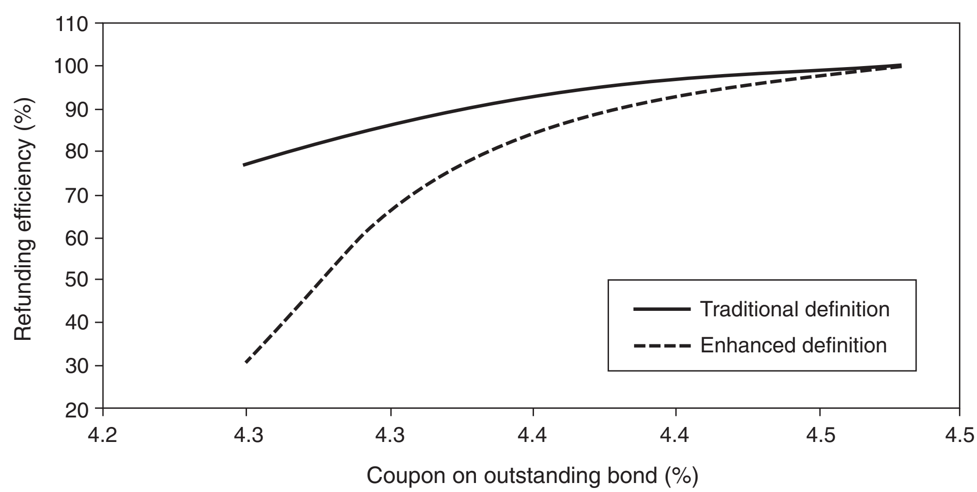

Fig. 5. Refunding efficiency of 5-year bonds refunded with maturity-matched callable 4.142% bond

Figure 5 compares the efficiencies shown in Figs 2 and 4. It shows that the 100% threshold occurs, as before, when the coupon of the outstanding bond is 4.48%. In other words, the optimal decision remains the same. On the other hand, below the 4.48% threshold the new efficiency declines much more steeply.

IV. Summary

In the textbook definition of refunding efficiency, the replacement bond is assumed to be non-callable. The question is how to measure efficiency when the replacement bond is itself callable. We have shown that the method currently used in the industry can lead to non-sensical recommendations. In particular, it can assign a high positive efficiency to transactions that are clearly losers. We have proposed a new formula that correctly measures refunding efficiency and have demonstrated the desirable properties of this new definition.

From a practical perspective, the significance of the new definition is that it always results in a lower efficiency than the one currently in use. Consequently it will often indicate that waiting is preferable to transacting. Using the new formula, issuers of callable bonds should be able to realize significant savings in managing their callable debt portfolios. Finally, the new definition is directly applicable to the refinancing decision for residential mortgages (Kalotay *et al.*, 2005).

References

Black, F., Derman, E. and Toy, W. (1990) A one-factor model of interest rates and its application to treasury bond options, *Financial Analysts Journal*, **46**, 33–9.

Black, F., Fischer and Karasinski, P. (1991) Bond and option pricing when short rates are lognormal, *Financial Analysts Journal*, **47**, 52–9.

Boyce, W. M. and Kalotay, A. J. (1979) Optimum bond calling and refunding, *Interfaces*, **9**, 36–49.

Finnerty, J. D. and Emery, D. R. (2001) *Debt Management: A Practitioner's Guide*, Harvard Business School Press, Boston, MA.

Howard, C. D. and Kalotay, A. J. (1988) Embedded call options and refunding efficiency, in *Advances in Futures and Options Research*, Vol. 3 (Ed.) F. Fabozzi, JAI Press Inc., Greenwich, CT, pp. 97–117.

Kalotay, A. and May, W. (1998) The timing of advance refunding of tax-exempt bonds, *Municipal Finance Journal*, **19**, 1–15.

Kalotay, A., Yang, D. and Fabozzi, F. (2005) When to Refinance a Mortgage, Working paper.