The Valuation and Management of Bonds with Sinking Fund Provisions

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Contemporary techniques for valuing a single interest-rate-sensitive option can be extended to the valuation of a package of interrelated embedded options, such as those contained in sinking fund issues. A typical public sinking fund issue may contain several options—a conventional call provision, a sinking fund acceleration feature, and a delivery option that allows the issuer to satisfy its sinking fund requirements through market purchases.

Analysis reveals that delivery and call options are nearly additive. That is, the coupon premium on a callable sinking fund bond (relative to the coupon on a comparable optionless bond) plus the coupon premium on a deliverable sinking fund bond will approximate the premium for a sinking fund bond with both call and delivery options. This is because the two options are exercised in opposite interest rate environments. Call and acceleration options, however, are largely redundant, because, in the presence of a call option, the acceleration provision is valuable only within a very narrow range of interest rate movements.

Distribution also affects the value of sinking fund bonds. Opportunistic investors can increase the value of an outstanding sinking fund issue by several points by accumulating a significant portion of the issue. At the same time, by making advance market purchases of sinking fund payments, the issuer can take defensive action against accumulators. These purchases reduce the issue's theoretical value by a quantifiable amount.

These findings have implications for the optimal management of sinking fund bonds by both investors and issuers.

Sinking fund bonds are perhaps the most complex of all fixed income securities because of the presence of interrelated **embedded** European and American **options**. In addition to a conventional call provision, sinking fund bonds typically include delivery, acceleration and designation options

Historically, sinking funds began as investment funds designed to accumulate gradually the assets necessary to retire debt at maturity. Now, the issuer of a typical sinking fund is required to retire a portion of the original principal on an annual basis, rather than in a single payment at maturity. Sinking funds have become a mechanism for retiring debt as the economic lifetime of the underlying assets diminishes, thus strengthening the credit of the issuer.[1]

Sinking fund bonds are ubiquitous in the global debt market. In the United States, they appear in the issues of industrials, gas pipelines, electric utilities and municipalities, as well as in preferred stock. Indeed, we estimate that 48% of all currently outstanding public U S. investment-grade corporate debt issues bear sinking fund provisions. Variations of the sinking fund structure are also found in Canadian debt, in Eurobonds (where they are particularly common in Yen-denominated issues) and in Dutch domestic debt.

The standard sinking fund provision, which is featured in this article, calls for a sequence of 20 equal payments of principal beginning in the 11th year of a 30-year bond. There are, however, many variations on this theme. For example, the sinking fund bonds of gas pipeline companies often have much shorter maturities, corresponding to the shorter economic lifetimes of the underlying assets. Other forms of sinking funds include the purchase fund, where the issuer is required to purchase a certain number of bonds in the open market each year, but only if the bonds are selling at a discount, and the invested sinking fund found in some Canadian debt, which is a fund of cash and certain permitted securities—including the underlying bond—set up to provide for the repayment of principal at maturity. This latter form is reminiscent of the origi-

nal spirit of sinking funds. Perhaps the most complex form of sinking fund is the funnel sinking fund found, for example, in the indentures of subsidiaries of the Southern Company, such as Georgia Power. There the company must retire a certain percentage of its total indebtedness annually, with the sinking fund being satisfied either by bonds from any of the company's issues or by certain unfunded property credits.

The options embedded in sinking funds afford the issuer substantial flexibility in the extinguishment of the debt. For example, the issuer may meet the mandatory sinking fund requirement either through cash payments at par or by delivering actual securities to the trustees. This so-called **delivery option** becomes important in periods of high interest rates, when the issuer can purchase bonds in the open market at prices below par.[2] During periods of low interest rates, the issuer can use the acceleration provision, which is a noncumulative option to increase the mandatory sinking fund payment by some multiple—in effect, a series of partial European calls at par.[3] At the same time, the issuer may wish to call all or a portion of the issue under the conventional call provision, often at a premium to par.

The distribution of outstanding sinking fund bonds also affects their value. The designation option allows the issuer to apply prepurchased bonds to future sinking fund requirements in an optimal manner, depending on the course of interest rates, so that prepurchases by the issuer tend to reduce the average value of the bonds that remain outstanding.[4] In complementary fashion, the accumulation of large blocks of bonds by investors reduces the value of the issuer's delivery option and, in turn, increases the value of the bonds.[5]

Our objectives in this article are to bring some order to the welter of options that decorate sinking funds, to establish the fair values for these options and to analyze their mutual interaction.

Option Valuation

Since the mid-1980s, major theoretical advances have been made in the valuation of interest-rate-sensitive options. These advances were motivated by, on the one hand, the explosive growth of the interest rate swap market and its corresponding puts and calls and, on the other, the large volume of calls and associated refundings of high-coupon bonds during 1986 and 1987. In spite of these advances, however, little progress has been made beyond the valuation of a single embedded option, such as a call or put.

The underlying valuation technology, which has become virtually standard, employs a so-called binary-tree structure and dynamic programming to determine the exact value of a bond. Dynamic programming allows optimal scenario-dependent actions by both issuers and investors to be internalized in the model. This approach is fundamentally different from the valuation of mortgage-backed securities, which is based on the direct simulation of interest rates and relies on externally supplied models of prepayment rates.

We have extended these contemporary option-pricing techniques to treat the multiple interrelated embedded options found in sinking fund issues, including puts. We examine the behavior of these options in new sinking fund issues and study the effect of distribution on value over the life of an issue. We conclude with some recommendations about the optimal management of sinking fund bonds for both investors and issuers.

Option Values in New Sinking Fund Bonds

As a concrete example of an outstanding sinking fund bond, consider the McDonalds Corporation sinking fund debentures of 2016, CUSIP 580135AS0. These bonds, $100,000,000 in face amount, were issued on February 28, 1986 to mature on March 1, 2016 They bear a coupon of 8.875% and pay interest semiannually. The bonds first become refundable on March 1, 1996 at a price of 104.050 and first become callable at par on March 1, 2006. The sinking fund calls for $5,000,000 of principal to be retired annually beginning March 1, 1997, either by cash payment at par or by delivery of actual securities to the trustee. The sinking fund provi-

Glossary

▶***Embedded Options:***
Options that are part of the structure of a debt security, as opposed to bare options, which trade separately from any underlying security.

▶***Delivery Option:***
An option giving the issuer the right to retire mandatory sinking fund payments through open market purchases. Without a delivery option, the issuer must meet all sinking fund payments by redeeming randomly selected bonds at par.

▶***Bullet Bonds:***
Bonds that have only a single scheduled principal payment at maturity; bullet bonds may have embedded options.

▶***Volatility:***
A measure of interest rate uncertainty that may loosely be regarded as the standard deviation of interest rates after one period, normalized by the level of interest rates.

▶***Yield-to-Worst:***
The cash flow yield under the cash flow scenario that is most detrimental to the holder of the security under the constant-interest-rate scenario.

sions also provide for a 150% acceleration, which is an optional redemption at par of an additional $7,500,000 of principal on each of the sinking fund payment dates.

The standard structure we study in this article is a 30-year bond with a level 5% sinking fund beginning in year 11. When the bond bears a call option, the bond will first be callable in year 10 at a price of par plus 60% of the coupon (i e, a bond with a coupon of 10% will initially be callable at 106). The call price will decline in equal annual increments until year 25. Thereafter, the bond will be callable at par.

Note that here we are using the notions of callability and refundability interchangeably. Technically speaking, many bonds are callable at or shortly after issue, although the issuer is precluded by the terms of the indenture from using the proceeds of lower-cost debt to retire the outstanding debt before it becomes refundable. For example, the McDonalds 8-7/8s of 2016 were callable at issue at a price of 108.100, but are not refundable until 1996.

Unless otherwise stated, all calculations in this article are made under the assumptions that the issuer's non-callable current coupon yield curve is flat at 8% and that the volatility of the short-term interest rate is 10%.[6]

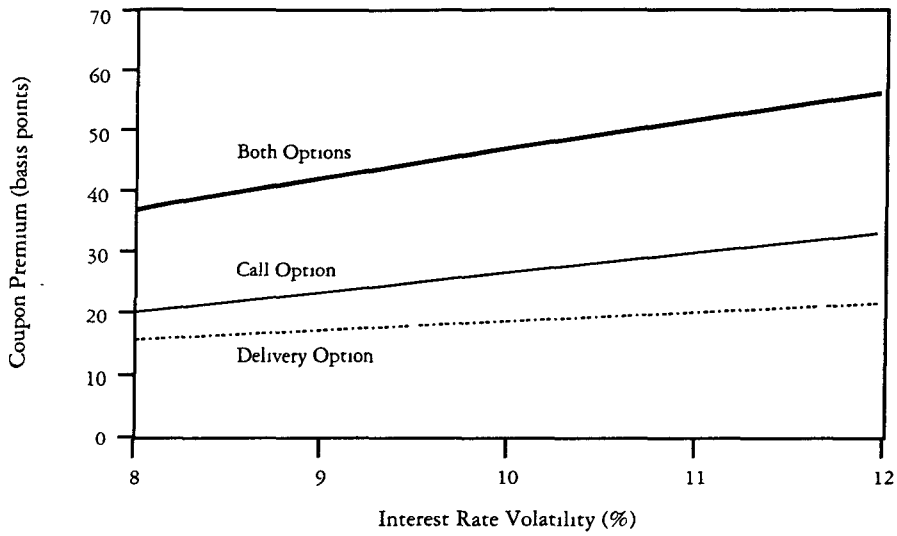
We study the various options embedded in new sinking fund structures in two ways. First, we determine the coupon premium over an optionless structure for each individual option or combination of options. Second, we determine the theoretical present value of those options.

We begin by determining the coupon at which the fair price of an optionless sinking fund bond is par, given the specified yield curve assumptions. Note that this coupon is independent of interest rate volatility. We then find the premium paid for a given option or package of options, given our assumptions of the term structure and volatility of interest rates, by subtracting the optionless coupon from the coupon at which the option structure under analysis is priced at par.

To determine the theoretical present value of the options embedded in an issue, we begin by determining the coupon at which the fair price of the issue is par. The value of the package of options is then the amount by which the value of the issue with the options "turned off" exceeds par. The stand-alone value of an individual option embedded in a bond is determined in like manner, but with par replaced by the value of the bond with all other options turned off. The coupon is held fixed throughout this analysis.

It is important to note the distinction between these analyses. In the first case, the premium for an individual option is determined by analyzing a bond bearing only that option; in the second case, individual option values are determined on the basis of a bond bearing multiple options. Note that this latter bond will have a higher coupon incorporating multiple premiums. As such, the value of any option embedded in this bond will differ from the value of the same option embedded in a bond bearing only that option. In general, larger coupons lead to greater values for options exercised in low-rate environments—such as a call or acceleration option—and to smaller values for options exercised in high-rate environments—such as a put or a delivery option.

Figure A Coupon Premiums for Call and Delivery Options as a Function of Interest Rate Volatility

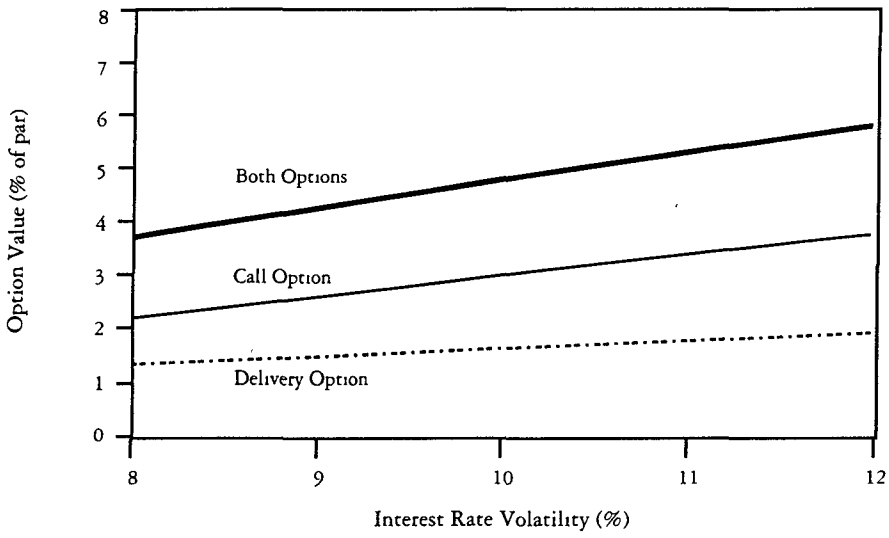


Call and Delivery Options

Because the volatility of interest rates is a driving factor in option pricing, we begin by considering the volatility-dependence of call and delivery options in new sinking fund issues. The issues we analyze here have no acceleration provision.

Figure A shows the premiums (in basis points) paid for these options relative to an optionless sinking fund bond, which in the flat 8% interest rate environment considered here carries a coupon of 8%. At a volatility of 10%, a fairly priced callable sinking fund bond would carry a coupon of roughly 8.27%, a deliverable sinking fund bond would carry a coupon of 8.19%, and the coupon on a bond bearing both options would be 8.47%. Under the same assumptions, the theoretical cou-

Figure B Present Values of Call and Delivery Options as a Function of Interest Rate Volatility



pon on a callable 30-year bullet bond is 8 46%, reflecting the longer average life of the bullet bond.

Figure B shows the volatility-dependence of the theoretical values of these options, both on an individual basis and in the aggregate. Here we analyze a single structure bearing both options. At a volatility of 10%, the call option on the 8.47% bond in isolation is worth 2.89 points and the delivery option is worth 1.68 points. When the interaction of these options is fully discounted, the aggregate value is 4.52 points, which is only 0.05 points less than the sum of the individual options.

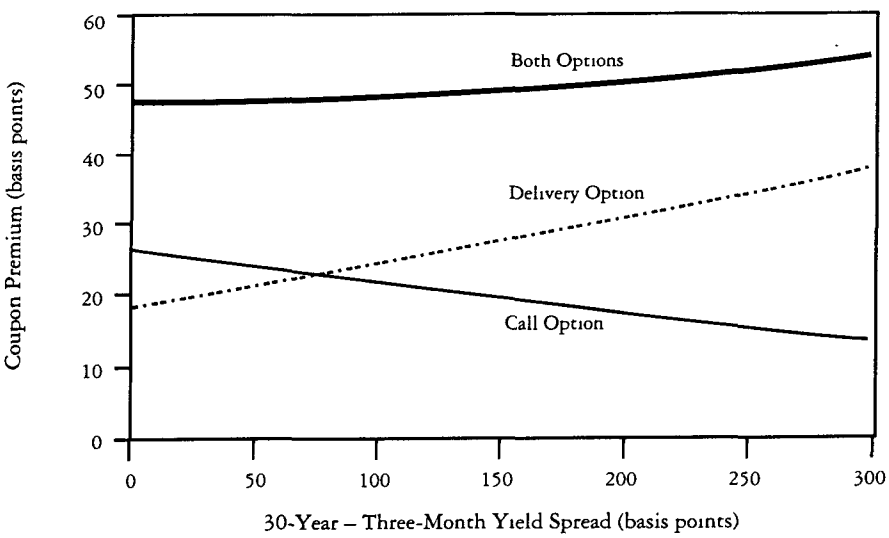
As these examples show, the coupon premiums and the option values are very nearly additive. In fact, additivity of premiums within a basis point holds over the entire range of volatility, while additivity of option values holds within 0.02 points at a volatility of 8% and within 0.11 points at a volatility of 12%. The slight departures from complete additivity arise because, even though the two options are exercised in opposite rate environments, they still interact with one another, partly through the additional coupon paid for the second option and partly through the forfeiture of the time value of one through the exercise of the other.

We again note that the individual option values shown in Figure B differ from those that would be measured in a bond priced at par and bearing only a single option, because that bond would have a lower coupon. In general, a larger coupon leads to a more valuable call option and a less valuable delivery option.

We can interpret an upwardly sloping yield curve as an expectation that interest rates will rise. Steeper yield curves should thus lead to more valuable delivery options and less valuable call options in new sinking fund structures. Holding the 30-year rate fixed at 8%, we swept the short-term rate through a 300-basis-point range from 8% to 5%. The remainder of the term structure was fitted to a Bradley-Crane model with no linear term.[7]

The coupon for an optionless sinking fund bond declines by about five basis points for every additional 50 basis points of spread between the long and short ends of the yield curve. This is because the distribution of principal payments over time leads to a coupon that is a discounted weighted average over the interest rate term structure. Figure C illustrates the premiums relative to that optionless base case for the individual and coupled options. Figure D shows the stand-alone and aggregate present values of the call and delivery options. As expected, the value of the call option declines and the value of the delivery option increases as the yield curve steepens; the delivery option becomes dominant when the spread between the long and short ends of

Figure C Coupon Premiums for Call and Delivery Options as a Function of the Yield Spread



the yield curve exceeds about 130 basis points.

Were Figure D a plot of the option values for bonds bearing but a single option, the call and delivery option curves would intersect at a spread of about 75 basis points, as the curves in Figure C do. Again, this is because the delivery option is more valuable and the call option less valuable, the lower the bond coupon. The effect in this case is on the order of a quarter point of value for each option over the entire range.

When the short end of the yield curve is at 5.5%, for example, the coupon on the optionless bond is 7.75%. In this case, the coupon on a callable bond is 7.92%, the coupon on a deliverable bond is 8.09%, and the coupon on a bond bearing both options is 8.26%. At the same time, the call option on the 7.92% bond is worth 1.65 points, while the call option on the 8.26% bond is worth 2.00 points. The delivery option is worth 3 41 points on the 8.09% bond but only 3.26 points on the 8.26% bond. The pair of options in aggregate is worth 5.13 points.

Call and Acceleration Options

As we have noted, an acceleration provision is essentially a sequence of European options to call a portion of the issue at par. An acceleration of 100%, called a double-up, is typical of many sinking fund issues. We estimate that 35% of all public U.S. sinking fund issues bear some form of acceleration provision and that fully 98% of those also bear a standard call provision. One would expect an acceleration provision to add little value to an existing call option, as the call is an American option on the entire issue, while the acceleration is a European option on only a portion of the issue.

Figure E illustrates the coupon premiums for call and acceleration options in a new sinking fund issue, given accelerations ranging from 0 to 500%. The uppermost curve is the premium paid for the combination of call and acceleration options. The premium for the call option in the absence of acceleration is given by the intercept of this curve, 27.5 basis points. The middle curve shows the premium paid for the acceleration provision in the absence of a call option, while the dotted curve at the bottom of the graph shows the almost negligible premium paid for an acceleration provision over and above a call option.

Figure F shows the theoretical values of the call and acceleration options over the same range of accelerations. The uppermost curve is the aggregate value of the call and acceleration options. The curve just below represents the stand-alone value of the call option. Just as the addition of a delivery option to a callable structure leads to an increase in the coupon and a concomitant increase in the value of the call option, additional acceleration leads to a slight increase in the call option's value.

Figure D Present Values of Call and Delivery Options as a Function of the Yield Spread

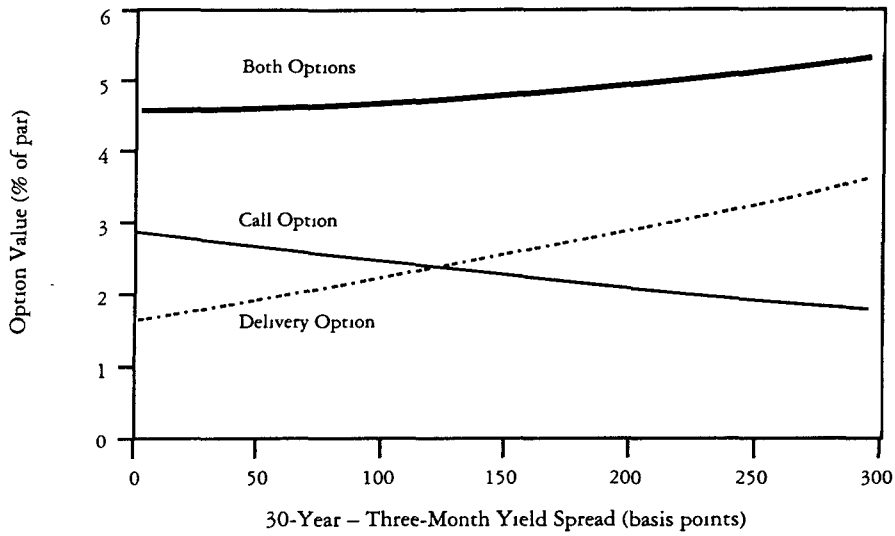


Figure E Coupon Premiums of Call and Acceleration Options as a Function of Acceleration

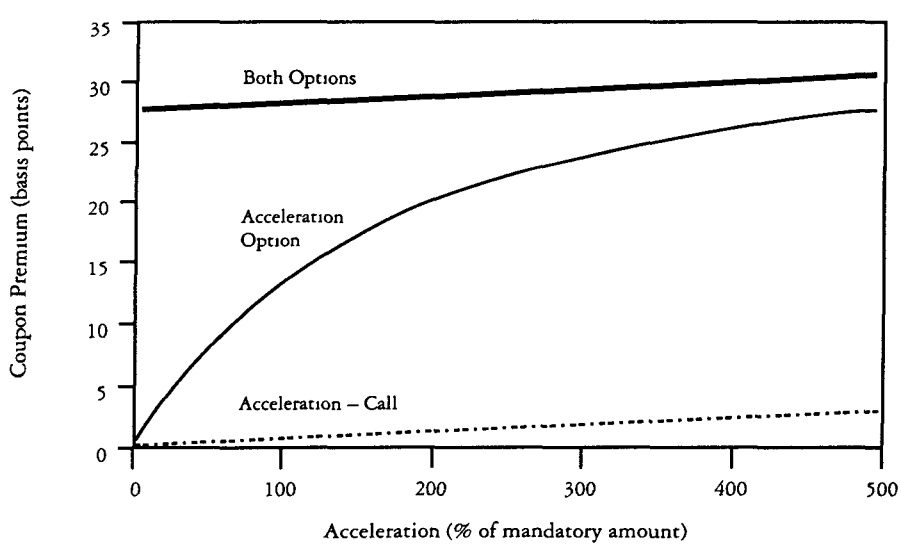
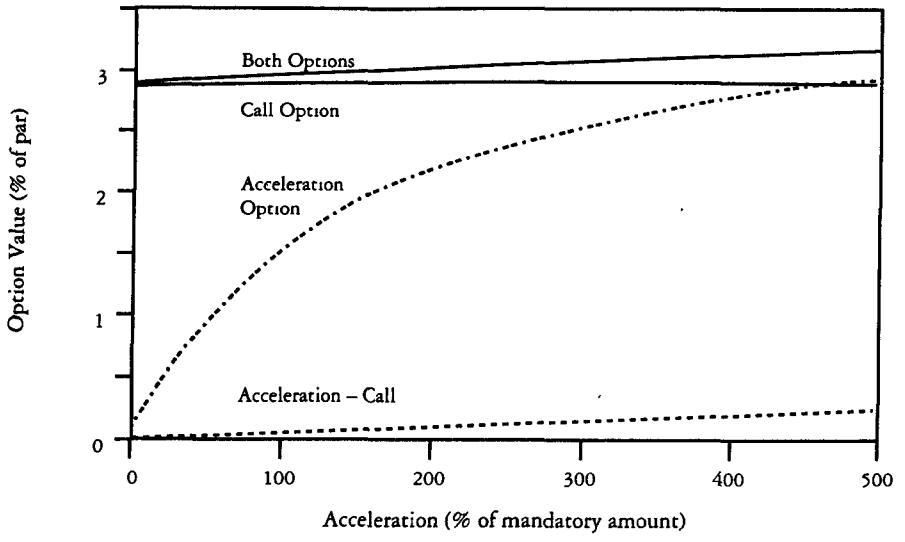


Figure F Present Values of Call and Acceleration Options as a Function of Acceleration



The dotted curve in Figure F illustrates the stand-alone value of the acceleration option. The value is slightly higher than it would be were the issue non-callable; again, this is because the premium paid for a call option raises the bond coupon. This effect is about 0.1 points at an acceleration of 200%. At larger accelerations, it becomes smaller.

Finally, the curve at the bottom of Figure F shows the value of the acceleration provision, given the presence of the call option. Clearly, an acceleration option adds little value to an existing call option. In large part, this is because the acceleration provision will be useful only within a narrow window—that is, when rates have fallen, but not sufficiently to offset the call premium.

An acceleration of between 400% and 500% is roughly equivalent to a standard call option in both cost and value. This point has apparently not been lost on some issuers. There are occasional non-callable issues with an acceleration of 200%—a triple-up option—such as the Nordstrom Credit 9-3/8s of 2016.

Distribution Effects

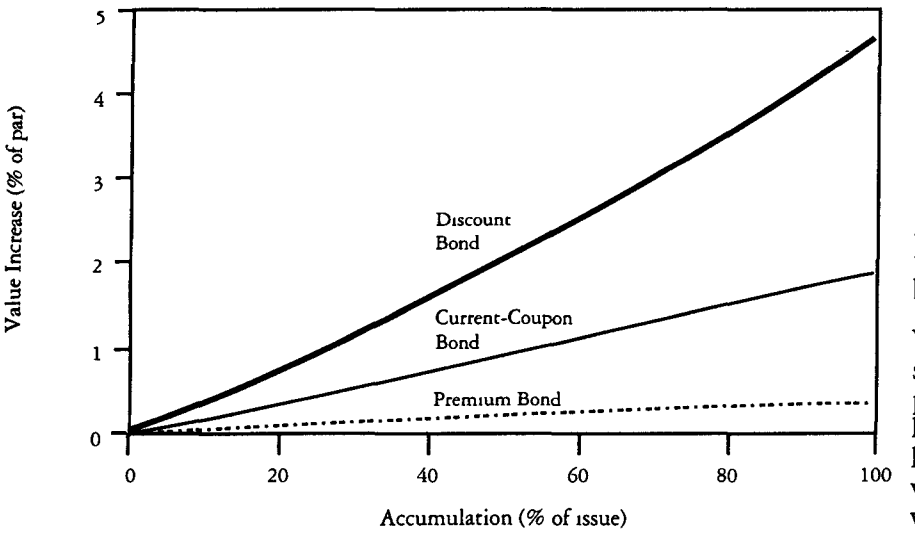
Because the cash flow characteristics of **bullet bonds**—those with a single maturity—are independent of the number of bonds outstanding and their distribution, the theoretical value of bullet bonds is unaffected by either issuer prepurchases or investor accumulation. In practice, liquidity considerations do come into play, with lower liquidity generally leading to lower market value. The theoretical value of sinking fund bonds, however, is profoundly affected by both issuer prepurchases and investor accumulation.

By collecting low-coupon bonds that would normally sell at a deep discount, investors can force the issuer to satisfy the sinking fund at par and thus enhance the value of the bonds. In the context of our model, this opportunity arises because the exercise of the delivery option requires that there be a willing seller in the open market, and collection effectively negates this possibility.

More generally, collection affects the value of all bonds, not just those selling at a discount, when interest rate volatility is taken into account. In the case of current-coupon bonds, those trading near par, the probability that rates will increase in the future lends significant time value to the issuer's delivery option. Indeed, as we showed above, the value of the delivery option at issue may equal or exceed that of the call. By canceling this option, the collector enhances the value of the bonds at the cost of the issuer.

We should point out that the issuer is vulnerable not just to the pernicious efforts of willful collectors, but also to the lack of liquidity produced by otherwise well-meaning loss-constrained investors. Thus, in assessing the

Figure G Value of Outstanding Sinking Fund Debt as a Function of Accumulation



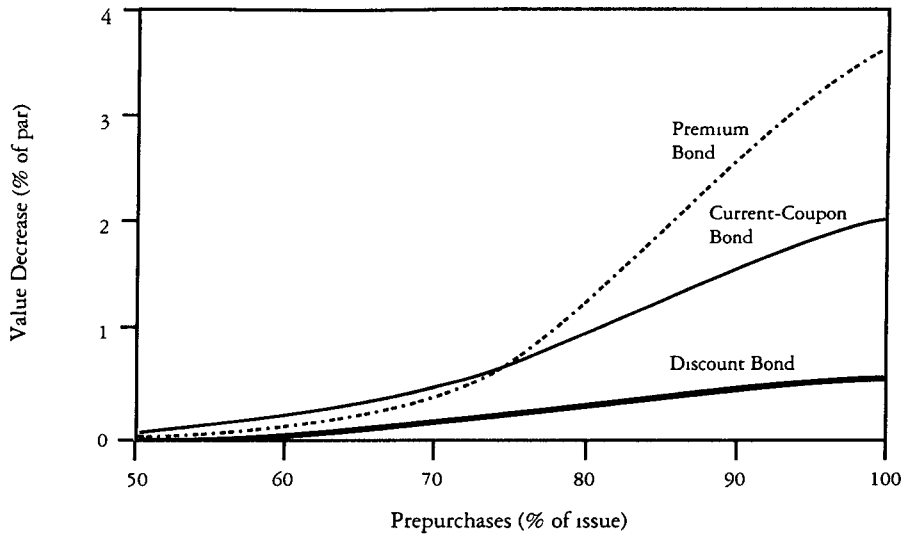
value of a sinking fund issue, all parties should consider both the float for the issue and the identity of its holders. In the analysis that follows, we lump both these effects into the category of accumulation [8]

Figure G illustrates how accumulation increases the value of outstanding sinking fund debt to the investor. Alternatively, this change may be viewed as the amount by which the issuer's delivery option decreases in value. The results presented are for year 10, one year before the sinking fund payments are scheduled to begin, and assume that the issuer has made no advance market purchases. The three bonds analyzed are a 7.25% discount bond priced at 90, an 8.80% current coupon bond and a 10.5% premium bond that would trade near its call price of 106.3, assuming no accumulation in any of these cases. Each of these bonds bears a delivery option and an American call option. None has an acceleration provision.

Not surprisingly, investors can gain a substantial amount of value by accumulating discount sinking fund bonds, while they appear to gain little from accumulating premium sinking fund bonds. These results bear out our earlier assertion that there is significant potential gain from accumulating current-coupon bonds, where the delivery option has strictly time value.

The delivery option quantifies the right of the issuer to buy bonds in the market to meet current sinking fund obligations. What are the implications of market prepurchases—the purchase of bonds in excess of current requirements—by the issuer? The answer to this question lies in the indenture. In some cases, the indenture may specify how the issuer must apply prepurchased bonds—to the front end of the sinking fund schedule, to the back end or, as is often the case in private placements, in pro rata fashion to the entire schedule.

Figure H Value of Outstanding Sinking Fund Debt as a Function of Prepurchase



But the indenture may grant the issuer more flexibility, either through a European designation option, which allows a designation of the issuer's own choosing at the time of purchase, or an American designation option, under which the issuer may hold the bonds in treasury and optimally choose the timing of their designation in a scenario-dependent manner.[9]

As the issuer acquires bonds in the market, the theoretical value of the remaining bonds will normally diminish. Because the magnitude of this effect depends upon the designation option, investors must be familiar with the terms of the indenture.

Figure H shows the decrease in value to the investor (or, conversely, the increase in value to

Figure I Value of Outstanding Sinking Fund Debt with Prepurchase and Accumulation

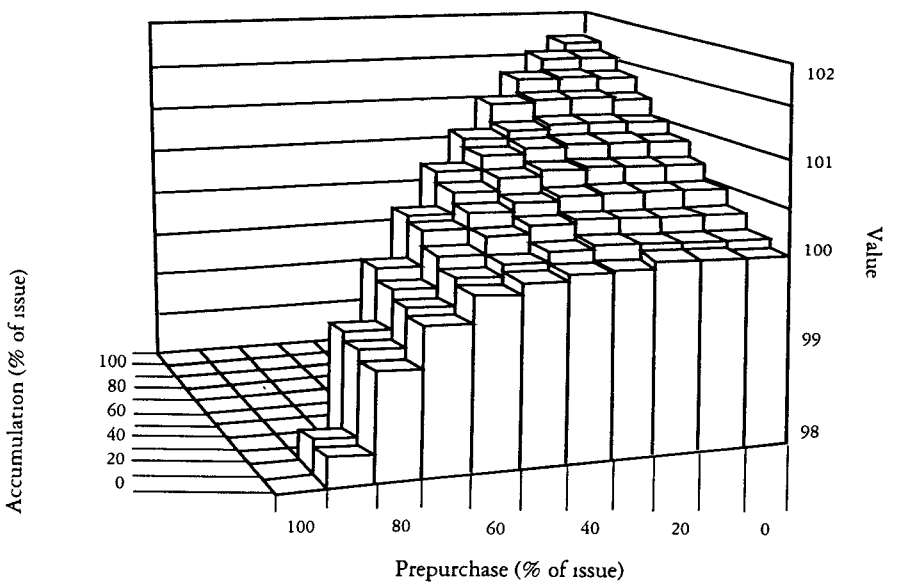
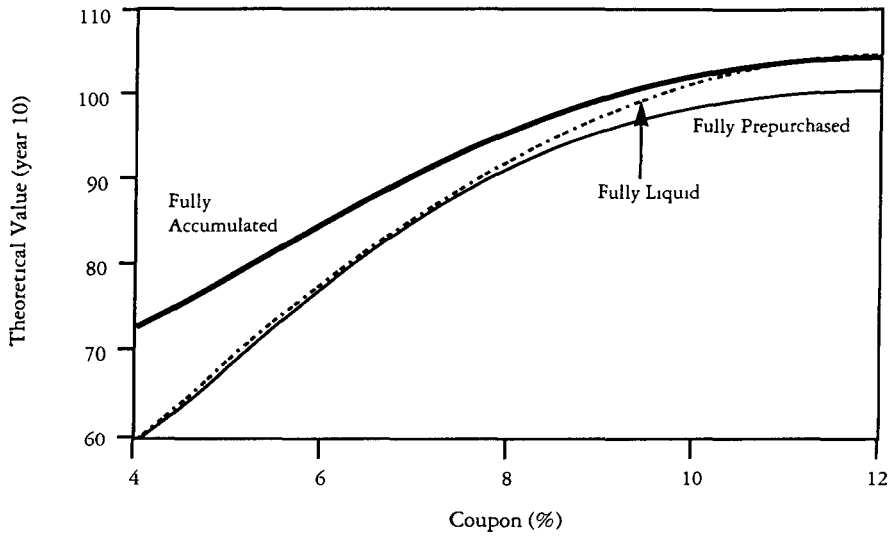


Figure J Range of Values of Outstanding Sinking Fund Debt and Bond Coupon



the issuer) due to issuer prepurchases, using the same three bonds illustrated in Figure G. The figure assumes no investor accumulation and an American designation option. Here, the premium bond is most affected, although significant value can also be extracted through the purchase of current-coupon bonds.

At first glance, it appears that the issuer must buy most of the issue to effect any significant change in value. However, issuer prepurchases can also prove useful as a defensive strategy against accumulators. As Figure G shows, the rewards reaped by accumulators are almost proportional to their combined holdings. Any prepurchases by the issuer act as a deterrent to potential accumulation by limiting its upside potential.

Figure I illustrates the full scope of the competition between issuer and investor. Here the value of the 8.80% bond analyzed above is displayed as a function of both issuer prepurchases and investor accumulation. What this surface reflects is that the market price of sinking fund bonds can vary in a highly dynamic manner as the distribution of the bonds changes, even in the absence of interest rate changes.

A corollary that may be drawn from these results is that the small investor is at a distinct disadvantage in the sinking fund game. Figure J shows the theoretical upper and lower limits of prices for 10-year-old sinking fund issues bearing both call and delivery options. These prices have been computed assuming the issuer's yield curve is flat at 8% and the volatility of interest rates is 10%. The dashed curve shows the value of the bonds assuming full liquidity (i.e., the entire issue is outstanding, there are no collectors and there are no loss-constrained investors). The range of theoretical prices is striking, being on the order of four points for bonds trading at 90 or above and growing even larger for deeply discounted issues. It is clear that the small investor risks dramatic changes in the value of his holdings; furthermore, the causes for these changes are not only beyond his control, but also completely independent of the usual sources of risk to bond investors—interest rate risk and default risk.

Management of Sinking Fund Issues

The presence of interrelated embedded options and the effect of distribution on theoretical value render sinking fund bonds perhaps the most complex of all fixed income securities from a management perspective. While analytical tools can be used to determine the fair value of a sinking fund issue under a specified distribution, the issuer and investors alike can alter that distribution, and in turn that theoretical value, via capital market transactions.

Our analysis of the package of options embedded in a sinking fund issue indicates that, in the presence of a conventional call option, the incremental value of an acceleration provision is minimal, and vice versa. The reason for this is that both these options become valuable when interest rates are low relative to the coupon, hence they are highly redundant The delivery option and the conventional call option, however, are nearly additive. This is because the delivery option is useful when rates are high and the bonds sell at a discount. Similarly, the combination of the delivery option with an acceleration provision provides the issuer with a formidable weapon.

We have considered in detail two practical problems—the structuring of new issues and the effects of distribution on outstanding bonds. While the examples presented are restricted to a typical 30-year issue, our methods can be applied to other structures.

We first determined the increase in coupon, relative to an optionless structure, due to the inclusion of various combinations of embedded options. This increase depends on the shape of the yield curve. As a rule, higher **volatility** results in higher incremental coupons. We estimate the coupon on a standard sinking fund structure to be about 40 to 50 basis points above the coupon on its optionless counterpart. We also find that the delivery option can be very valuable, particularly when the

yield curve is upward sloping. In fact, under normal conditions, the value of the delivery option on a new issue approximately equals the value of the call option. Empirical investigation suggests that the market places a disproportionate weight on the call option relative to the delivery option.

From an issuer's point of view, managing a call option on a sinking fund bond is quite different from managing a call option on a bullet bond.[10] With a bullet bond, the choice is to call either all or none of the issue; with a sinking fund bond, a partial call is often optimal. The called bonds are normally designated to the back end of the sinking fund, and the bonds that have been left outstanding are brought in at par through mandatory and optional sinking fund payments. Should the issuer choose to exercise the conventional call provision, however, the uncanceled bonds held by it or its subsidiaries will be subject to pro rata call if it fails to designate these holdings against specific sinking fund dates.

Continuing with the valuation of new issues, we find that even the initial distribution of the bonds can affect their average value by as much as two points (although we have not demonstrated this here). Specifically, investor accumulation can increase the price by almost two points, while issuer prepurchases can reduce the value of the remaining bonds by more than half a point.

The effects of distribution can be more pronounced on outstanding issues close to the commencement of the sinking fund. The most dramatic effects of accumulation come when the coupon is low relative to the prevailing interest rate. But even a current-coupon bond's value may be increased by nearly two points via accumulation.

These findings have several implications for managers. Small investors need to recognize that large block trades expose them to considerable risk. They need to be extremely conservative and value sinking fund bonds on a "**yield-to-worst**" basis. In other words, they should be prepared to hold the bonds until maturity if rates are high and, at the same time, expect to be the first ones to be sunk out at par if rates are low. This is because the issuer may repurchase a large fraction of the issue without the participation of the small investor.

Large investors, on the other hand, can and should be opportunistic. By acquiring large blocks at the outset, they can effectively preempt the issuer. The accumulation of current-coupon bonds, in particular, may provide a substantial defense against rising interest rates

At the same time, issuers should also be active buyers of their own bonds, possibly even shortly after issue. Prepurchase of discount bonds offers an immediate defense against accumulators, while prepurchase of current-coupon bonds discourages accumulators. Finally, prepurchase of premium bonds can be a cost-saving alternative to the exercise of the conventional call provision.

Footnotes

1 *See, for example, R S Wilson and F J Fabozzi,* The New Corporate Bond Market. A Complete and Insightful Analysis of the Latest Trends, Issues and Advances *(Chicago Probus Publishing Company, 1991), pp 188–197 and references therein*

2 *A J Kalotay, "On the Management of Sinking Funds,"* Financial Management, *Summer 1981 and A J Kalotay, "Sinking Funds and the Realized Cost of Debt,"* Financial Management, *Spring 1982*

3 *A J Kalotay and P McIntyre, "Sinking Fund Issues A Challenge for the Active Manager" (Salomon Brothers Inc, New York, November 1984) and C D Howard and A J Kalotay, "Embedded Call Options and Refunding Efficiency,"* Advances in Futures and Options Research *3 (1988), pp 97–117 In rare instances, sinking fund payments are at a slight premium to par*

4 *A J Kalotay, "Sinking Fund Management" (Paper presented at the Western Finance Meetings, March 1987) and A J Kalotay and B Tuckman, "Sinking Fund Prepurchases and the Designation Option" (New York University Salomon Center, 1990)*

5 *K B Dunn and C Spatt, "A Strategic Analysis of Sinking Fund Bonds,"* Journal of Financial Economics *13 (1984), pp 399–423, B Tuckman, "The Accumulation Game in Sinking Fund Bond Issues' (Massachusetts Institute of Technology, 1988), and Kalotay, "On the Management of Sinking Funds,"* op. cit.

6. *Option values are determined using a proprietary one-factor arbitrage-free yield curve model in which the short-term interest rate (the one-month rate in this study) is lognormally distributed with a volatility that is constant over time. The model is designed to match exactly the current term structure of interest rates For the purpose of computation, the model is used to generate a binomial tree in the short-term interest rate Cash flows are discounted back along the branches of the tree at the appropriate short-term rate Discussions of option valuation models of this type can be found in B Black, E Derman and W Toy, "A One-Factor Model of Interest Rates and its Application to Treasury Bond Options,"* Financial Analysts Journal, *January/February 1990, J Hull and A White, "Valuing Derivative Securities Using the Explicit Finite Difference Method,"* Journal of Financial and Quantitative Analysis *25 (1990), pp 87–100, J. Hull and A. White, "Pricing Interest-Rate Derivative Securities,"* Review of Financial Studies *3 (1990), pp 573–92, and J Hull and A White, "One Factor Interest-Rate Models and the Valuation of Interest-Rate Derivative Securities" (unpublished)*

7 *S P Bradley and D B Crane, "Management of Commercial Bank Government Security Portfolios An Optimization Approach under Uncertainty,"* Journal of Bank Finance, *Spring 1973*

8 *See Kalotay, "On the Management of Sinking Funds,"* op. cit.

9 *See Kalotay, "Sinking Fund Management,"* op. cit. *and Kalotay and Tuckman, "Sinking Fund Prepurchases and the Designation Option,"* op cit

10 *See Kalotay and McIntyre, "Sinking Fund Issues,"* op cit.