An Analysis of Original Issue Discount Bonds

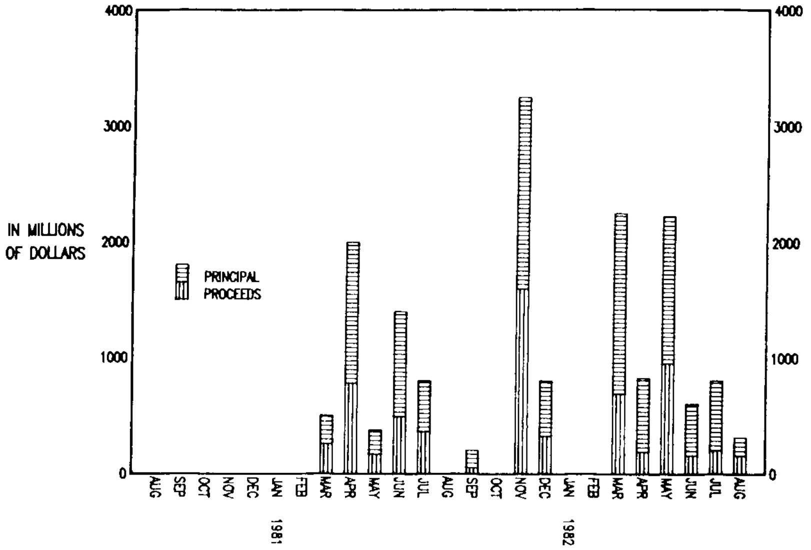
Andrew J. Kalotay

Mr. Kalotay, an Associate Editor of this Journal, is Vice President of Salomon Brothers in New York City.

1. Introduction

Although the total amount of outstanding original issue discount debt (OID) is not readily available because many of these issues were marketed overseas, we do know that as of September 1982 public domestic issues alone exceeded $7B in proceeds and $17B in principal amount, as shown in Exhibit 1. The first public OID was sold in March 1981 (by Martin Marietta) and was followed in April by the first zero coupon bond (by J.C. Penney). Some OIDs, however, were privately placed about a year earlier [10]. Prior to 1980 OIDs were virtually nonexistent, although due to unusual circumstances occasional issues had in fact been priced substantially below par.

Exhibit 1. Volume of Domestic OID Bonds, August 1980–August 1982



A recent change in the tax treatment of the discount, requiring scientific rather than straight line amortization, has had the effect of reducing the supply of new OIDs. Scientific amortization of the discount eliminates a previously existing tax incentive for issuing OIDs by corporations. However, OIDs retain features that will be attractive to certain issuers and investors. Stripped treasury securities, such as CATs, provide investors with an obvious alternative to OIDs, and the recent proliferation of strips needs no comment. Moreover, it continues to be of considerable academic interest to explain the sudden popularity of OIDs in 1981 and 1982. Why were OIDs not introduced much earlier? In this paper we shall provide convincing evidence that *the interaction of the old tax treatment of the discount and the historical high level of interest rates was responsible for this phenomenon.*

For the issuer, the obvious alternative to an OID is a conventional par bond (P). Comparison of an OID to a P necessitates a reexamination of the traditional call and sinking fund features, the significance of corporate income taxes, the adequacy of standard summary statistics such as internal rate of return for the cost of debt securities, and the appropriate study horizon.

An OID may appeal to the issuer for two related reasons. First, on a pretax basis the cost of an OID is usually significantly lower than that of a P. Depending upon the issue's maturity and the prevailing market environment, bond professionals estimate the pretax savings to range from 50 to 150 basis points. This is consistent with the statistical findings of [8], according to which issuers saved, on the average, about 100 basis points in 1981 and about 75 basis points in 1982. On certain issues, possibly due to novelty effects, the savings were significantly greater. The pretax savings are believed to be primarily attributable to the reduced interest rate risk provided by an OID to the long-term investor. For all practical purposes an OID is fully call-protected. Moreover, to the investor the reinvestment risk of interest payments is reduced, or, in the case of zero coupons (Zs), completely eliminated. It has also been suggested that the pretax savings are influenced by the shape of the yield curve [19]. According to this model the yield spread between fully call protected current coupon bonds and OIDs will be wider when the yield curve is downward sloping, as in 1981–82, due to the longer effective life of the OIDs. This model has not been tested empirically to date.

The second source of attraction of an OID to the issuer has been the straight-line amortization of the discount. Under this treatment the taxable equivalent cost[1] of an OID is lower than its pretax cost [5, 10, 17], while the taxable equivalent cost of a P is the same as its pretax cost. The change from the straight-line to the scientific (constant yield) method which is now required eliminates this tax incentive.

The appeal of an OID has been recognized for a long time. In particular, the tax benefit to the issuers was pointed out by Racelle and Lewellen [18] in 1976, while the need for a long-term debt security which provides a guaranteed return to the investors can be traced to a 1971 paper by Fisher and Weil [6]. Kaufman [14], in 1973, suggested that the U.S. Treasury issue long-term Zs. Given this long-standing recognition of the benefits provided by Zs, perhaps the most surprising fact is that OIDs were *not* issued prior to 1980.

We shall consider OIDs from the viewpoint of the issuer, emphasizing interest rate risk and tax effects. The special problems associated with the early retirement of an OID will be discussed in Section 2. The pretax yield advantage of an OID over a P can be partially attributed to the full *de facto* call protection of the former. Clearly the issuer can reduce the nominal cost by selling fully call-protected par bonds instead of OIDs.

In Section 2, we estimate the incremental yield associated with the standard call feature on a P, and show that it accounts for most of the yield differential between OIDs and Ps. The early retirement of OIDs creates unique problems not only because they are call protected, but also because they cannot be issued conveniently with a sinking fund. The section closes with a discussion of the latter problem.

In Section 3, we look at OIDs on an after-tax basis. We establish that under the straight-line amortization of the discount the taxable equivalent cost of an OID is lower than its pretax cost, and that this cost advantage is significant only in a double-digit interest rate environment. This provides an explanation of why OIDs were not sold prior to 1980. The section then presents a review of the academic literature of OIDs and closes with a numerical illustration of the relevant considerations.

Since the traditional IRR computation is inadequate for comparing the cost of a current coupon bond and an OID, in Section 4 we introduce two orthogonal models to quantify the effect of changing interest rates upon the cost of an issue. One approach is to assume that the issuer funds all intermediate payments and repays the

[1]The taxable equivalent cost is defined as the aftertax cost divided by (1 − tax rate).

accumulated total at maturity. According to this "future value" method, on an after-tax basis the risk of a Z is that rates decline, while the risk of a P is that rates rise. Motivated by the future value method, we show that after-tax duration can exceed an OID's maturity. The second approach for quantifying interest rate risk is to compute the cost of funds over a horizon which is beyond the maturity of the issue. In this context the cost of financing with an OID is more sensitive to the "rollover" rate than the cost of financing with a P.

2. Refunding Considerations

a. Incremental equilibrium yield of standard call feature

As the coupon rate of an OID is substantially below the prevailing level of interest rates, for all practical purposes an OID is protected from refunding at the customary par call price. Thus, disregarding extreme aberrations of the yield curve, an OID should always yield less than a similar quality conventional callable P. An important question is *how much* of the yield spread between an OID and a P can be attributed to their different call features.

Since long-term corporate bonds usually carry standard call features (five-year initial call protection for finance companies and public utilities, ten-year call protection for industrials, with call premium gradually declining from a large fraction of the coupon to zero), the incremental yield associated with the call feature is difficult to estimate empirically. The existence of this incremental yield has been recognized, but without reference to specific estimates. A notable exception is [2], which, using option valuation techniques, provides a methodology for estimating the yield spread.

The fundamental assumption in [2] is that the logarithm of long-term interest rates follows a random walk without a trend; thus the standard deviation is the only parameter required to specify the process. The yield curve is assumed to be flat. The investor is assumed to be tax-exempt, while the issuer is assumed to have a marginal tax rate of approximately 50%. For a specified bond, the issuer's optimum refunding policy is determined by stochastic dynamic programming. Given the issuer's optimum refunding policy, the expected value of the bond to the investor is computed. In this manner the upper and lower bounds of the coupon on a new issue are determined. Above a certain level the issuer would find the cost of the call provision (which is in the form of a coupon premium) unacceptably high, and below a certain level the investor would not be compensated adequately. However, as is shown in [1], due to tax differentials there is a range that is acceptable to both parties.

As discussed in [2], the results derived by this methodology are surprisingly consistent with expert opinion. For example, in 1977 in an 8% interest rate environment the cost of the standard call feature on a long-term bond with five-year call protection was estimated to be between 25 and 45 basis points. Rates at that time were much less volatile than today. In the following exhibits we shall update these estimates to reflect current market conditions.

Exhibit 2 displays the minimum and maximum yield premia, associated with the standard "Bell System type" call structure, over a range of maturities in a 15% interest rate environment.[2] According to this exhibit, in a 15% environment the issuer whose marginal tax rate is 50% should be willing to pay over 100 basis points on a long-term bond for the call feature, while the non-taxable investor should be willing to accept as little as 40 basis points. The 70 basis point average of these extremes provides a heuristic estimate of the actual yield spread. The relative insensitivity of the spread to the maturity of the bond is due to the dependence of the call premium on maturity. For example, since the call price five years prior to maturity drops to par, the initial call price of a ten-year bond is 100, while the initial call price of a 40-year bond is almost 113.

Exhibit 3 shows the sensitivity of the average yield premium to rate volatility. Option pricing specialists without exception consider 0.07 to be too low, but there is less unanimity about the upper bound of 0.13.

[2]The volatility of the process (*i.e.*, annual standard deviation of the logarithm) is assumed to be 0.10. Thus, if current rates are 15% the 95% prediction interval one year away is from 12.28% to 18.32%.

Exhibit 2. Incremental Yield of Callable Bonds, 15% Interest Rate Environment, 0.10 Volatility

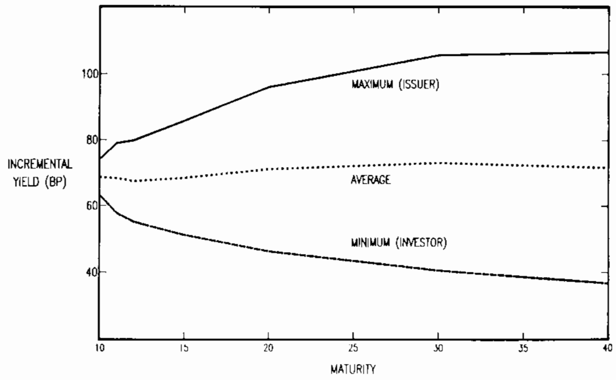
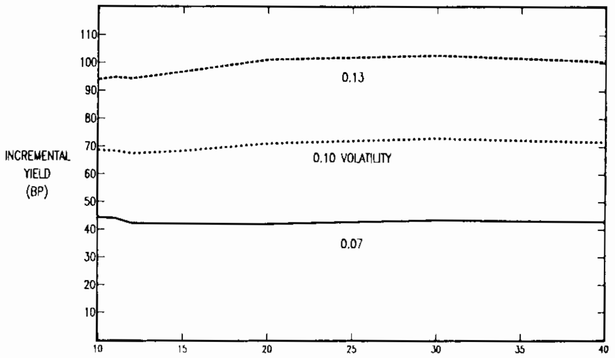


Exhibit 3. Estimated Incremental Yield of Callable Bonds, 15% Interest Rate Environment



The greater the volatility, the higher is the corresponding yield premium. According to Exhibit 3 the cost of the standard call feature on a long-term bond is between 50 and 100 basis points. This result appears to be consistent with the opinion of most bond professionals. It is also interesting to note that according to [8] the average yield spreads between a P and an OID were about 100 basis points in 1981 and about 76 basis points in 1982 when rates were generally lower. Thus, the spread can be basically attributed to superior refunding protection of OIDs.

b. Early retirement of OIDs

Several methods have been proposed for restoring the callability of an OID, and certain municipal OIDs have already been issued with strong call provisions. The usual method is to set initial call price below par and then escalate the call prices back to par. Another method of making an OID refundable is Leibowitz's index call concept [15]. In this case the call price, which is based upon the prevailing market rate, is designed to be higher than the market price of a similar but fully call-protected bond. Boyce's work on tender offers for non-callable bonds incorporates a similar idea [3].

Since industrial bonds usually carry sinking fund provisions, it may seem surprising that industrial OIDs are always issued as "bullet" maturities or as serial bonds. The reason is that sinking fund OIDs cannot be structured conveniently.

This can be seen by considering the *ex post,* rather than the *ex ante,* cost of a sinking fund issue. As discussed in [11], the *ex post* cost depends upon the prices at which the issuer repurchases the bonds. The maximum price that the issuer ever has to pay for a bond to satisfy the sinking fund is 100% of the face value, which happens to be about equal to the proceeds generated by a conventional issue. It may be less if the issuer is able to purchase the bonds in the marketplace at a discount. Thus the cost of a conventional issue will not exceed the coupon rate, which happens to be a reflection of the interest rate environment at the time of issuance, and may be less. Obviously it would be desirable to have the same cap on the cost of an OID.

However, there is a fundamental problem with any publicly held discounted sinking fund obligation, and a sinking fund OID would be no exception. The problem is that the entire issue might be purchased by a single investor ("collector"). Once the bonds are collected, the issuer would be obliged to purchase the sinking funds at par, irrespective of interest rate movements [11]. For this reason, as we shall see, a sinking fund OID cannot be priced easily.

There are two extreme ways of pricing of sinking fund OID: on a yield-to-maturity (YTM) basis or on a cash flow yield (CFY) basis. In the former case the price is such that the resulting yield to an investor who holds the bonds to maturity is competitive with the prevailing market rates. In the latter case the issue is priced essentially as a private placement, the fundamental assumption being that the sinking fund payments are made (on a *pro rata* basis) at par and the imputed CFY is competitive with prevailing market rates. Note that if an issue is sold at par, the two yields are identical. However, in the case of OIDs the YTM method results in lower prices than the CFY method.

If a sinking fund OID were priced on a yield-to-maturity basis, then the price would be relatively low, and the issue would be susceptible to collection. If collected, the *ex post* cost to the issuer would be unacceptably high. On the other hand, if the bonds were priced on a cash flow yield basis, then the price would be higher. In this case the return to an investor who held the bonds to maturity would be unacceptably low. Without assurance of realizing a reasonable return, no individual investor would buy a part of the issue. Of course, an investor could purchase the issue in its entirety and realize the nominal cash flow yield, but that arrangement essentially would be a private placement.

The only way of resolving this dilemma is to allow the issuer to repurchase bonds for sinking fund purposes at prices accreted according to the original yield to maturity. That would assure that each investor realizes a fair return, although the life of any individual bond would still be uncertain. Considering that the primary attraction of a Z to the investor is its guaranteed return over a specified horizon, a sinking fund Z would lose much of its attraction. The easiest way for the issuer to stagger the maturities is by selling serial OIDs rather than sinking fund OIDs. In fact, such serial OIDs have already been issued.

3. Tax Considerations

The nominal cost advantage of an OID over a P can arise from two sources. First, due to its call protection, an OID has a lower cost even on a pretax basis. This consideration was discussed in Section 2. Secondly, under the straight-line amortization of the discount formerly permitted, the taxable equivalent cost (TEC) of an OID was lower than the pretax cost, while the TEC of a P was and remains equal to its pretax cost (see footnote 1).

The straight-line amortization of the discount for tax purposes was probably a result of an arbitrary accounting decision — there appears to be no obvious financial justification for it. Clearly the faster the discount recognized for tax purposes, the more attractive are the after-tax cash flows. Given the arbitrariness of straight-line amortization, we would not expect the pretax and taxable equivalent costs to be equal under this method. Our intuition tells us that the amount of deduction at any time should be roughly proportional to the market value of the issue — less around the time of sale, more close to maturity. The so-called scientific method of amortization follows precisely such a pattern.

The claim that under straight-line amortization the taxable equivalent cost is lower than the pretax cost can be shown by a dominance argument. Instead of the straight-line method we assume that the discount is amortized scientifically in computing the proportion of the discount that represents deductible interest, *i.e.*, that the original proceeds are accreted over time to the principal amount, the rate of accretion being determined by the pretax cost and the coupon. In that case the taxable equivalent cost is the same as the pretax cost, as shown in [12]. The cost under straight-line amortization is obviously lower than under scientific amortization, since the tax deductions are advanced while all other flows are unchanged.

In order to eliminate this tax incentive for issuing OIDs, the method of amortization was changed to the scientific (constant yield) method.[3] This treatment was retroactive to July 1, 1982. The legislation changing the method was signed by the President on September 3, 1982.

The benefit of straight-line amortization to taxable issuers, and the appetite of investors for long Zs have long been recognized [6, 18]. And yet, not until 1981 were OIDs sold in significant quantities. Why, one might wonder, was this extremely desirable innovation not introduced much earlier? In spite of Livingstone's claim to the contrary [16], the demand for Zs has always existed. The reason for the absence of OIDs was not the lack of demand.

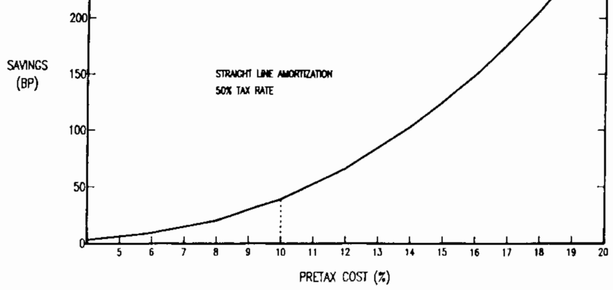
Corporate treasurers, who are generally conservative, tend to be reluctant to experiment with new financing instruments. Their aversion to risk dictated that OIDs be introduced *gradually,* for obviously no issuer could be expected to suddenly shift from a P to a long-term Z.

The natural course of innovation dictated that either the maturity of the new instrument be relatively short or its coupon be reasonably high, in order to keep the discount moderate. Unfortunately, when the discount is moderate so are the resulting tax savings, while the perceived risks of a completely novel financing instrument are substantial. It was the double-digit interest rate environment that eventually enabled, if not mandated, issuers to overcome their initial skepticism.

Exhibit 4 substantiates this claim. The exhibit displays the savings from a ten-year Z, under straight-line amortization, over a range of yields. Evidently, the savings are barely observable in a 6% environment, and not very impressive even when rates are at 8%. However, in a double-digit environment the savings begin to appear significant, as they increase essentially exponentially.

Exhibit 5 provides similar analysis for a 30-year "bullet" issue priced at 50 (this determines the coupon). Once more, only very high interest rates generate significant savings. Finally, Exhibit 6 depicts the long-term Treasury rates over time. Exhibits 4, 5 and 6 provide a convincing rationale for both the nonexis-

Exhibit 4. 10-Year Zero Coupon Bond, Pretax Cost – Taxable Equivalent Cost



[3] Canadian tax laws already required scientific amortization.

Exhibit 5. 30-Year OID (Proceeds = 0.5 * Principal Amount; Pretax Cost − Taxable Equivalent Cost)

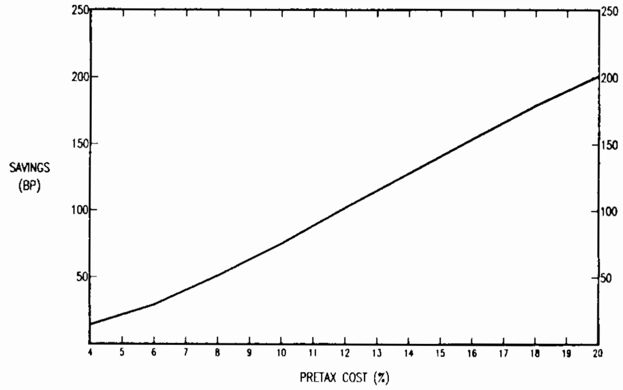
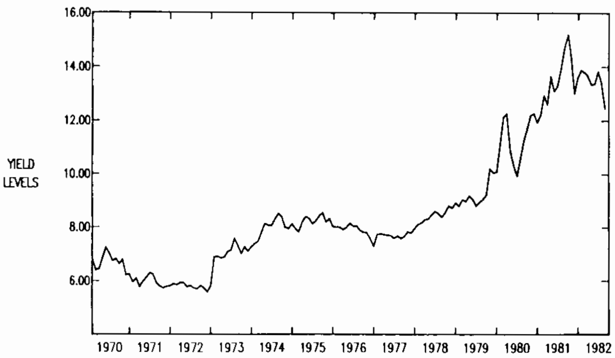


Exhibit 6. Historical Monthly Closing Levels for U.S. 30-Year Treasury Bonds



tence of OIDs prior to and their emergence after 1980.

As we shall see, the OID concept has been around for a long time, and the fundamental validity of this innovation has never been seriously challenged by corporate treasurers. But until fairly recently interest rates were so low that the savings from an OID would have been insignificant, even under straight line amortization. Moreover, the non-refundability of an OID always entails a certain element of risk *vis-à-vis* a conventional issue. In the absence of a precedent, prior to 1981 there was also considerable institutional uncertainty regarding the accounting, tax, and regulatory treatment of an OID, not to mention the possibly adverse response to such a revolutionary instrument by the rating agencies. However, once interest rates climbed well into double digits, the risk/return tradeoff of selling an OID became more easily justifiable.

At this point, it is instructive to review the pre-1980 academic thinking regarding OIDs. It is particularly noteworthy that the relevance of tax considerations has been explicitly recognized by virtually everyone, but evidently there was considerable confusion about institutional aspects of the bond market.

The seminal paper by Fisher and Weil [6] emphasized the reinvestment risk of a coupon-bearing bond to an investor with a specified long-term return requirement. This problem is not generically tax-related. To enable the investor to cope with reinvestment risk, Kaufman, in 1973, proposed that the U.S. Treasury sell long-term Zs [14]. Without questioning the validity of Kaufman's idea, we note that, because of tax considerations, corporations would have been more suitable candidates than the U.S. Treasury for selling Zs. Kaufman, in fact, was well aware of the relevance of taxes to OIDs. The potential investors he envisaged were taxable individuals, rather than tax-exempt institutions. The motivation for his proposal was that, in the special case of U.S. Treasury bonds, investors would not be obliged to amortize the discount over the life of the issue, but could pay the tax at maturity. This, of course, is the precise treatment of Series E savings bonds.[4] Kaufman's idea was that the Treasury could actually reduce its borrowing cost by selling Zs to taxable individuals, and his proposal has considerable merit even today.

The correct tax treatment of a corporate OID was described in 1976 by Racelle and Lewellen in [18]. Evidently, by that time the benefits of an OID to both buyer and seller were well recognized. However, the tax treatment of municipal bonds sold at a discount is still not entirely transparent.

Livingstone's 1979 note [16] could have been a major setback to the eventual emergence of OIDs, as he proved that Zs would never be issued. He correctly argued that a Z has a negative value for an investor in a high marginal tax bracket, but never considered pension funds, retirement accounts, foundations, or investors not subject to US tax regulations, such as Japanese residents, as potential purchasers. The obvious flaw of [16] is that it assumes essentially symmetrical tax parameters for investor and issuer. Even though retrospectively [16] appears to be somewhat naive, the complete mishandling of tax considerations is, unfortunately, not an isolated incident in academic publications.[5]

As we have discussed in Section 2, OIDs are essen-

[4]Series E savings bonds are essentially putable Zs.

[5]The dialogue between Ang [*Journal of Finance*, June 1975, and *Journal of Finance* 1978] and Mayor and McCoy [*Journal of Finance*, March 1978] concerning the profitability from the refunding of discounted debt is an excellent illustration. Even though this transaction is usually motivated by tax considerations, the analyses were inevitably carried out pretax. Mayor and McCoy actually "proved" that discounted debt can never be profitably refunded, providing a complement to Livingstone's result that Zs would never be sold.

tially fully call-protected. Traditional corporate bonds are normally refundable in 5 or 10 years. The curious interaction between refunding provisions and tax parameters is investigated in [1]. The primary result of [1] is that, due to their different tax rates, both investor and issuer prefer a callable bond to a non-callable one.

The callable bond will command a higher coupon than its non-callable counterpart, but on an after-tax basis the expected value of the call option exceeds its cost. This particular tax incentive is lost when an issue, such as an OID, is fully call-protected. Of course, straight-line amortization provides an unrelated and more significant tax incentive.

In summary, the following factors need to be considered in comparing the nominal cost of a conventional callable P with an OID:

(1) on a pretax basis, the OID's lower cost can be attributed to its reduced interest rate risk, as manifested primarily by its call protection and secondarily by the reduced reinvestment risk of the coupon payments;

(2) under straight-line amortization the taxable equivalent cost of an OID is lower than its pretax cost;

(3) on an after-tax basis, a callable bond is preferable to its fully call-protected current coupon counterpart.

As an extreme example, consider the alternatives of issuing a 30-year P with standard call features at 15% versus a 30-year Z at 14.25%. Under straight-line amortization the taxable equivalent cost of the Z is 13.14%; under the compound interest method it is 14.25%. In contrast, the cost of P depends upon the course of interest rates. However, the estimated value of the call feature is roughly 100 basis points. Thus, the Z has a lower cost, on an expected value basis, only if the discount can be amortized straight-line. An interesting possibility to be explored is the restoration of the call on a Z, as discussed in Section 2. The call feature will increase the nominal yield level, but its value should exceed its cost.

4. Cost Measures

The traditional measure of the cost of a debt security is its after-tax cash flow yield, *i.e.*, the internal rate of return of the after-tax flows. According to this measure a non-callable issue is not subject to interest rate risk. Curiously, issuers do not necessarily prefer a Z to a non-refundable P having the same maturity but a significantly higher cash flow yield. Evidently, issuers envisage some interest rate risk, even though such is not directly encompassed by the traditional definition of cost.

The after-tax cash flow pattern of a Z is fundamentally different from the cash flow pattern of a P. The tax-deductibility of interest expense merely reduces cash outflows in case of a P, but the amortization of the discount actually generates cash inflows in the form of tax savings over the entire life of a Z. Thus the issuer of a Z can be perceived to be an investor in future tax savings. If he can reinvest these savings at a sufficiently high rate, the terminal value of the investment at maturity could exceed the face amount of the issue resulting in a negative cost. In contrast to an OID, a conventional bond generates inflows, pretax or after-tax, only at the time of the sale.

To manifest the issuer's notion of interest rate risk, we shall develop two approaches. One approach is to compute the cost on a future value basis by assuming that all intermediate cash flows are funded at the prevailing rate: that is, coupon payments net of tax savings are borrowed or invested, so there are cash outflows to be borrowed in the case of Ps and OIDs, cash savings to be invested from Zs. We then calculate the resulting all-encompassing terminal obligation. We define cost in the future value sense as the taxable equivalent of the annual rate which equates the initial proceeds to this terminal payment. According to this definition the cost of a P *decreases,* while the cost of a Z *increases,* as rates decline.

The second approach is to examine the cost of funds over a horizon which is beyond the maturity of the issue. Here all maturing principal is "rolled over" at the prevailing rate for the remainder of the horizon [11]. The cost of the issue can be summarized by the present value or the internal rate of return of the resulting flows. More generally, the "future value" approach and the "extended horizon" approach can be combined in the obvious way. As we shall see, under the extended horizon approach the cost of a Z, as determined by the cash flow yield, is more sensitive to future rates than the cost of a P. This, however, may not be the case when cost is defined in the "future value" sense.

"Future Value" Cost

The "future value" approach may be appropriate as long as the firm's incremental debt financing exceeds its after-tax interest payments, *i.e.*, as long as the growth rate of outstanding debt exceeds the average after-tax cost. In that case we can argue that the firm finances its interest expense by selling additional debt. How is the "future value" cost of a Z affected by interest rates? First, we note that since on an aftertax basis a Z does generate intermediate cash flows, its eventual cost is uncertain. Secondly, we observe that the cost of a Z actually *increases* as rates decline, since the earnings on the tax savings are reduced.

Exhibit 7 displays the "future value" cost of a ten-year non-callable P and a similar Z, with identical 13% pretax costs,[6] over a range of reinvestment rates. As shown in this exhibit, for sufficiently high rates the "future value" cost of a Z will actually be negative.

Inspection of Exhibit 7 suggests that the issuer can hedge interest rate risk over a ten-year horizon by selling the appropriate mix of P and Z. In fact, he can achieve virtually the same result by selling an OID whose after-tax interest payments are precisely offset by the tax savings generated from the amortization of the discount under straight-line amortization [17]. However, a mix of Z and a P would normally be superior to a single OID, partly because of the incremental tax advantage associated with making the P callable, and partly because the individual components of the portfolio would appeal to diverse groups of investors.

After-tax duration

The "future value" method suggests that we formally evaluate the after-tax duration of a fixed income security. Although this notion is not new [7], it is noteworthy that under the scientific method of amortization of the discount the general definition collapses to a closed-form solution, as shown in [13]. The result is reproduced in the Appendix, for the sake of completeness.

[6]In order to make the analysis strictly comparable, we assume that the discount on the Z is amortized scientifically, so that the after-tax internal rates of return are also identical.

The after-tax duration of a 10-year bond at a constant pretax 14% yield, over a range of coupon rates, is displayed in Exhibit 8. As the coupon declines, the duration increases and eventually exceeds the maturity date. (Under straight-line amortization the duration would be the same as the maturity at the point where the after-tax interest payments are exactly offset by the tax savings from the amortization of the discount.)

The observation that, on an after-tax basis, an OID's duration can exceed the bond's maturity has an interesting practical implication for the issuer. An issuer cannot "lock in" the prevailing rate over a specified horizon by selling a par bond of the appropriate duration. The reason for this is that since the maturity of a par bond *exceeds* its duration, the eventual liquidation of the bond prior to maturity will give rise to tax-related complication. For example, if the bond is obtained for cash, the resulting gain or loss is taxable, and under certain circumstances the tax is amortized over several years. However, the issuer can "lock in" the prevailing rate by selling a Z of the required duration instead of a P.

Extended horizon method

As we discussed earlier, a P and a Z generate fundamentally different cash flow patterns. In the "future value" method above we were concerned with the risk entailed by the reinvestment/borrowing of the intermediate cash flows prior to maturity. We shall show below that the principal payment can also be a potential source of risk. Corporations tend to fund principal payments by selling additional debt in the required amount at the interest rates prevailing at the time of maturity. Since the principal payment of a Z tends to

Exhibit 7. Rollover Future Cost to Taxable Issuer, 10-Year Par *versus* 10-Year Zero, 10-Year Horizon

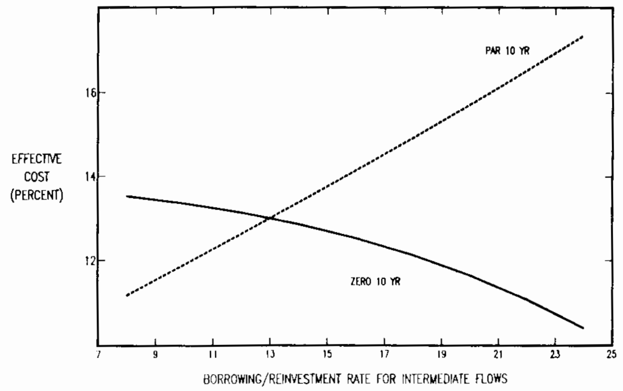
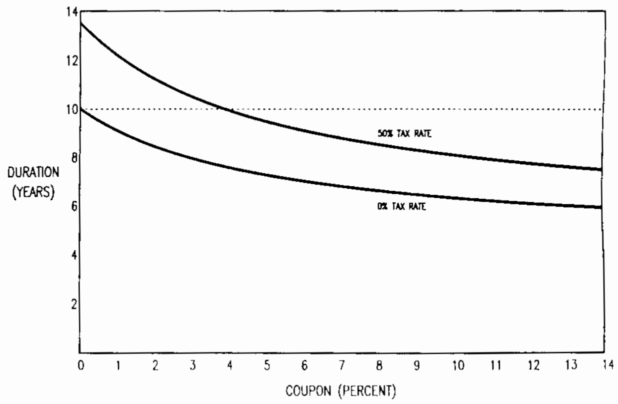


Exhibit 8. Ten-Year Bond Yielding 14%



be several times larger than the principal payment of P generating the same proceeds, the interest rate risk of bridge financing with a Z is going to be greater than bridge financing with a P.

In our second approach we assess the interest rate risk of an issue by considering the carrying cost of funds over a horizon beyond the issue's maturity. Exhibit 9 displays the carrying cost of the ten-year P and the ten-year Z, each yielding 13%, over various horizons, and under three "roll-over" interest rate assumptions. If cost is measured by IRR, then the interest rate risk of the Z is greater than that of a P.

5. Summary

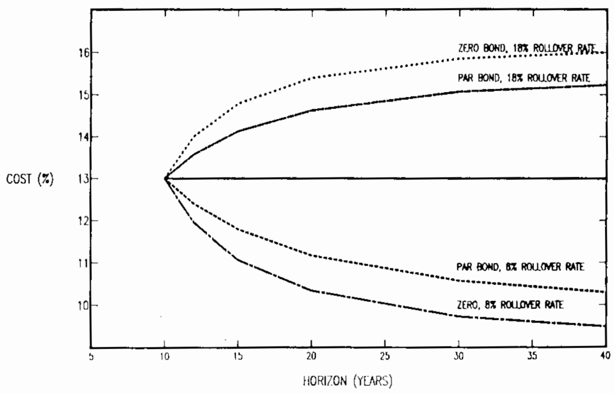
The emergence of OIDs has been one of the most exciting developments in the history of the fixed income markets. The cash flow pattern of an OID differs drastically from that of a conventional security, and the traditional call and sinking fund provisions are not readily applicable.

We have estimated that the call protection provided by an OID instrument should reduce its yield by about 70 basis points below that of a conventional issue. The absence of reinvestment risk is worth considerably less.

On an after-tax basis, the previously allowed straight-line amortization of the discount resulted in a further cost reduction, but this effect was significant only in a double-digit rate environment, where it can exceed 100 basis points. When comparing a P and a Z, we must also recognize that the tax adjusted value of the call provision exceeds its incremental cost.

Under the assumption that the issuer funds all intermediate cash flows, we have analyzed the interest rate risk of various securities. In contrast with a conventional issue, whose cost increases as rates increase, the after-tax cost of Z actually decreases. In a related result we established that, on an after-tax basis, the duration of an OID can actually exceed the bond's maturity. Finally, we suggested that, in order to assess the interest rate risk, cash flows should be considered over a horizon that extends beyond the issue's maturity.

Exhibit 9. Taxable Equivalent Cost, 10-Year 13% Par Bond *versus* Zero Coupon Bond, Alternative Rollover Rate



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Appendix. Aftertax Duration Under Scientific Amortization

Consider an OID in face amount 1, coupon c, maturity N sold at price p. We denote the pretax flow yield (IRR) by y and the investor's marginal tax rate by t.

Under the scientific method, A_k, the amount of discount amortized during the k-th period, can be shown to be

$$(1+y)^{-(N-k+1)}(y-c),$$

resulting in a cash flow of $-tA_k$ at the end of the period. The aftertax cash flow yield Y_t is $(1 - t)y$. (Under straight-line amortization the relationship between the pretax and after-tax yields is more complex.)

The aftertax Macaulay duration D_t is defined by

$$\frac{\sum_{k=1}^{N} k\ f_t(k)\ (1+y_t)^{-k}}{\sum_{k=1}^{N} f_t(k)\ (1+y_t)^{-k}}.$$

where $f_t(k)$ represents the after-tax cash flow at the end of period k. In the special case of an OID D_t reduces to

$$\frac{(1-t)cF(x) - t(y-c)(1+y)^{-(N+1)}F(x') + Nx^N}{P} \qquad \text{(A-1)}$$

where

$$x = (1+y_t)^{-1}, \quad x' = (1+y)x,$$

and

$$F(z) = \sum_{k=1}^{N} kz^k \begin{cases} = z(1-z)^{-2}\,[1-(N+1)z^N + Nz^{N+1}] & \text{if } z = 1, \\ = N(N+1)/2 & \text{if } z = 1. \end{cases}$$

We note that if $t = 0$, (A-1) collapses to the familiar Macaulay formula.