Applied Financial Economics Letters, 2008, **4**, 141–149

Optimal mortgage refinancing: application of bond valuation tools to household risk management

Andrew J. Kalotay[a], Deane Yang[b] and Frank J. Fabozzi[c,*]

[a]*Andrew Kalotay Associates, Inc., 61 Broadway, Suite 1400, New York, NY 10006*
[b]*Polytechnic University, Six Metrotech Center, Brooklyn, NY 11201*
[c]*Yale School of Management, 135 Prospect Street, New Haven, CT 06520-8200*

Despite the enormous volume of refinancing activity in conventional residential mortgages, reaching record levels during recent years of historically low interest rates, the solution to the problem of how to time refinancing decisions optimally has remained elusive. It is recognized that the decision should depend, among other factors, on the 'call' options of the outstanding and the new mortgage. Determining the value of these options is a challenge in the absence of an observable optionless mortgage yield curve. We solve this by calibrating a benchmark interest rate process to the value of the new mortgage and then apply the notion of refinancing efficiency to the timing decision. In particular, risk-averse decision makers can use refinancing efficiency to measure how close to optimal a refinancing is. We analyse the sensitivity of the decision to interest rate volatility and also show how to incorporate homeowner-specific considerations, namely borrowing horizon and income taxes. While calibration and refunding efficiency are well-known techniques in bond analysis, there is no evidence, hitherto, of their application to the mortgage-refinancing problem.

Research on household risk management involving mortgage financing has focused on two analytical issues. The first is the borrower selection of the optimal contract design (see Campbell and Cocco, 2003). While at one time only fixed-rate, level-payment mortgages were available, with maturity being the only variable that could be selected by the borrower, today there are a good number of mortgage designs from which to choose – fixed rate, adjustable rate and hybrid adjustable-rate products.[1]

Regardless of design, home mortgages in the US typically permit homeowners to prepay at any time without penalty.[2] This prepayment option is often exercised when rates decline in order to refinance more cheaply. The analytical challenge is to determine the optimal time to do so. This is the second area of focus in the research and the subject of this article. While there are several excellent theoretical articles that address the problem of how to time a refinancing decision optimally, to our knowledge

*Corresponding author. E-mail: FABOZZI321@aol.com
[1] For a review of these mortgage designs, see Fabozzi (2006).
[2] While there are prepayment penalty mortgages available, their origination is a small part of the mortgage market.

Applied Financial Economics Letters ISSN 1744–6546 print/ISSN 1744–6554 online
http://www.tandf.co.uk/journals
DOI: 10.1080/17446540701493729

there is no single one that offers a workable, useful method for borrowers. There is also considerable related research focusing on the effect of the prepayment option in the valuation of mortgage-backed securities. Work of this genre initially appeared in the 1980s and has proliferated since the 1990s.[3] Notably, none of the research, to our knowledge, takes any cognizance of the vast body of work on bond refunding.

In this article, we show that well-established principles for optimal bond refunding can be adapted to the mortgage-refinancing question. Standard option pricing theory tells us that a loan should not be refinanced until the savings realized equals the value of the call option. However, borrowers may not want to wait until the absolutely optimal moment. In fact, without the tools introduced in this article, they would not even be able to determine what is 'optimal'. Homeowners who refinance early do not extract the full option value. On the other hand, if they wait for rates to decline further when the option is in the money, they continue to pay above-market rates. Clearly homeowners need assistance to determine the optimal time to refinance and to measure how much money they leave on the table if they decide to refinance sub-optimally.

In Section I, we review the mortgage refinancing literature and then discuss the use of refunding efficiency. The original Boyce–Kalotay definition of efficiency assumes that the new loan is noncallable; we show how to extend it to the case where the new loan is itself callable. The calculation of savings and option values requires an optionless yield curve. While such a curve is available for corporate treasurers, an optionless mortgage yield curve cannot be observed. In Section II, we show how to infer it by calibrating an interest rate process using a benchmark curve to current mortgage rate levels. Using the calibrated curve, we calculate the corresponding refinancing efficiency of long-term fixed-coupon mortgages. In Section III, we explore how volatility impacts the refinancing decision and in Section IV consider how to incorporate homeowner-specific considerations, namely borrowing horizon and personal income taxes, into the decision. We summarize our findings in Section V.

I. Review of the Mortgage-Refinancing Literature

Greater awareness among borrowers of mortgage rate levels has increased the volume of refinancing activity in recent periods of declining mortgage rates.[4] But the borrower faces a dilemma. The risk of waiting until the mortgage rate drops to a desired level is that it may take a long time to do so and in the interim the homeowner continues to pay interest at an above market rate. The risk of refinancing too soon is that rates continue to decline and repeated refinancings incur additional transaction costs.

In the absence of an accepted scientific method, people rely on rules of thumb. With regard to the refinancing of a long-term fixed coupon mortgage, old-timers tend to refer to the 2% rule: refinance when the new rate is 2% (200 basis points) below the current rate. But times change – instead of 2%, today people tend to think of half a percentage point. An obvious reason for the lower threshold is that nowadays refinancing is more convenient and transaction costs are significantly lower than what they used to be decades ago.

In contrast to mortgage refinancing, the problem of optimal bond refunding has received considerable academic attention. Boyce and Kalotay (1979a) proposed a rigorous option-based approach that has become accepted and widely used today. As we will see, their notion of *refunding efficiency* can be adapted to mortgage refinancing. Refunding efficiency is the ratio of savings (expressed throughout this article in present value terms) to the value of the call option. While risk-neutral borrowers will 'pull the trigger' when and only when efficiency reaches 100%, those who are risk-averse may want to act earlier.

A mortgage is in essence a callable amortizing bond and the refinancing decision faced by homeowners is similar to the bond refunding decision faced by corporate treasurers whose firms have issued callable debt. Like a risk-averse treasurer, a homeowner may not want to wait until the optimal moment to refinance, so a measure of optimality such as refunding efficiency is a valuable tool for homeowners as well. The refinancing efficiency of a mortgage explicitly accounts for interest rate uncertainty. It enables homeowners to measure how

[3] See, for example, Archer and Ling (1993, 1997), Deng (1997), Deng *et al.* (2000), Hilliard *et al.* (1998), Kariya and Kobayashi (2000), Kau *et al.* (1992, 1995), Kau and Slawson (2002), LaCour-Little (1999), Stanton (1995), Stanton and Wallace (1998), Bennett *et al.* (2000), Hoover (2003), Chinloy (1991), Longstaff (2002), Le Roy (1996), Roncoroni and Moro (2005), Pliska (2005) and Krainer and Marquis (2003).

[4] In fact, in prepayment modelling there is an effect called the 'threshold-media effect'. It is so named because there is a barrage of both general media reporting and mortgage banking advertisements when mortgage rates drop to historically low levels, making borrowers aware of the refinancing opportunities.

close a refinancing is to being optimal and play the odds as effectively as a corporate treasurer.

There are several articles that address the optimum refinancing problem from the perspective of the individual homeowner, rather than from that of an investor. It is also recognized that individual concerns, such as expected length of stay and personal income taxes, should be considered. The existing research, however, does not demonstrate in practical terms how to incorporate the relevant factors into the refinancing decision. Siegel (1983) shows that the optimal strategy is to always refinance if the call option is in money. Chen and Ling (1989) explain that the refinancing decision depends on a comparison of the current call option and the value of the future compound option to refinance. Pliska (2005) provides an interesting theoretical model for optimal refinancing using Markov decision chains, but does not discuss implementation. Hoover (2003) presents a break-even equation-based model that incorporates some of the relevant factors, however it omits several important ones.

Let us begin by summarizing the problem. The relevant variables include the terms of the outstanding mortgage, the terms of the refinancing offerings and transaction costs. In addition, as mentioned above, there may be borrower-specific factors, such as expected length of stay and personal income taxes.

The decision, as several references point out, should depend on the relationship of the achievable cash flow savings to the loss of optionality. As some authors observe, the loss of optionality is the difference between the forfeited option of the outstanding mortgage and the acquired option in the new mortgage. While these observations are correct, at the time of their publications they were already well known in the bond refunding literature. Almost 30 years ago, Boyce and Kalotay (1979a) discussed these same considerations, including the need to incorporate the value of the call option of the new loan, as is the case with mortgages.

Boyce and Kalotay also discussed suboptimal option exercise and the need to measure the degree of suboptimality. They defined the refunding efficiency of a callable bond assuming that it would be refunded with a maturity-matched noncallable bond[5] as follows.

$$\text{Efficiency}_{\text{original}} = \frac{\text{Cash flow savings}}{\text{Forfeited option value}} \qquad (1)$$

Here ‘Cash flow savings’ is the present value of the differences in the cash flows less transaction cost and ‘Forfeited option value’ is the value of the call option embedded in the outstanding loan. Since the latter always exceeds (or is equal to) the former, which is the intrinsic value of the option, the efficiency cannot exceed 100% and is equal to 100% when and only when it is optimal to refund. This definition has become the standard for the bond refunding decision (see, for example, Finnerty and Emery, 2001).

Let us turn to the case where the new loan is itself callable. When future cash flows are interest rate dependent, the analysis becomes more complicated, because the possibility of future refunding must also be accounted for. It is well understood that refinancing is optimal when

$$\text{PV(Cash flow savings)} = \text{Net loss of optionality}$$

But how should we measure the degree of suboptimality in this case? In other words, how should we define efficiency when the new loan is callable?

As mentioned above, Boyce and Kalotay actually discussed this problem but did not provide an explicit formula. The option of the refunding issue can be incorporated into the efficiency formula as (Kalotay *et al.*, 2007):

$$\text{Efficiency}_{\text{new}} = \frac{\text{Change in cash flow value}}{\text{Change in option value}} \qquad (2)$$

Note that if the new loan is optionless, formula (2) collapses to formula (1). It is also shown that the efficiency of any plausible refunding transaction cannot exceed 100%. As with the original definition, efficiency is equal to 100% if and only if it is optimal to refund. The results are obtained using formula (2).

In implementing an efficiency-based refinancing policy, we need to set a threshold for exercise. A risk-neutral homeowner should refinance when and only when, the efficiency is 100%. Note that postponing refinancing when the efficiency is 100% entails a substantial ‘cost of carry’ (because the rate of the outstanding mortgage is much higher than the prevailing market rate), but it does not increase expected savings. In contrast, when the efficiency is below 100% the expected upside more than offsets the cost of carry.

As we mentioned above, a risk-averse borrower may want to refinance even before efficiency reaches 100%. While the acceptable level is a personal decision, we do not recommend refinancing below 95% efficiency.

[5] Howard and Kalotay (1988) employed the same approach in a more accessible article.

II. Mortgage Refinancing Efficiency

Next we tackle the refinancing decision in a realistic setting. But if we attempt to apply formula (2) directly, we note that there is a missing piece of information. Unlike in the case of bond refunding, where the borrower's optionless yield curve is known, a noncallable mortgage yield curve cannot be observed. Rates for new mortgages of different terms are available, but they always include the call option.

Another critical difference between bond refunding and mortgage refinancing is the homeowner's 'borrowing horizon'. If a corporation wants to retire its bonds prior to maturity, it can repurchase them in the market at a price close to their theoretical fair value. However for a generic homeowner 'fair value' is virtually meaningless. If the homeowner moves, he must repay the par value of the outstanding, rather than its fair value, i.e. the principal amount that he owes, rather than the fair value of the principal. Therefore the borrowing horizon of a homeowner who plans to move five years from now is five years, independent of the terms of the mortgages under consideration. We will examine the effect of the borrowing horizon in Section III; for now, we assume that the borrowing horizon coincides with the maturity of the mortgage.

The basic information about the new mortgage is its interest rate, maturity and discount points (here assumed to be zero). In addition, because the new mortgage is callable, in order to determine the value of the option we have to make an assumption about the transaction cost associated with possible refinancing in the future – throughout this article assumed to be 1% of the remaining principal.

In the absence of an optionless mortgage yield curve, we choose a benchmark curve and calibrate it to the new mortgage. The benchmark yield curve should be reasonably well correlated to mortgage rates; the obvious candidates are the Treasury yield curve or the swap curve, such as in Table 1. By 'calibration' we mean calculating the option-adjusted spread (OAS) such that the value of the mortgage equals par (see, for example, Kalotay *et al.*, 1993). Calculating the OAS of a security to a benchmark, such as the swap curve or the Treasury curve, is a standard tool in the bond market. It is a measure of relative value and it is particularly useful when the borrower's optionless yield curve is unavailable. Curiously, we could find no reference in the mortgage refinancing literature to the use of OAS. In the absence of an optionless mortgage yield curve and without the use of the OAS technology, it is hard to see how one could develop a workable approach for the mortgage refinancing decision.

Before illustrating the calibration process, we observe that the value of a mortgage depends on the perspective of the observer. On the one hand, the market determines the fair interest rates for new mortgages based on the expected prepayment behaviour of large groups of borrowers (Kalotay *et al.*, 2004). But each borrower should value a mortgage from his particular perspective. This is accomplished by calculating the OAS of the new mortgage based on borrower-specific cash flows (based on his horizon and tax considerations). Specifically, we need to determine the OAS that equates the value of the mortgage to its proceeds (normally par) and then use the resulting OAS to value the outstanding mortgage.

Note that in the case of bonds, where the treasurer knows the optionless yield curve, there is no need to calibrate. But even in that case, although the alternatives are fair from the capital market's perspective, borrower-specific considerations may introduce a bias. For example, it has been shown that taxable corporations should prefer callable bonds to optionless bonds (Boyce and Kalotay, 1979b). In the case of tax-exempt bonds, an issuer should use the taxable rate, rather than the tax-exempt rate, to value its liabilities (Kalotay and Tuckman, 1999).

Let us return to the refinancing problem. Suppose that a homeowner is deciding whether to refinance a 5.75% 30-year mortgage with a new 5.50% 30-year mortgage. The initial step is to calculate the OAS of the new mortgage against the benchmark yield curve given in Table 1. This calculation requires that we specify the transaction cost associated with refinancing the new mortgage in the future, should an opportunity arise; we assume that this cost is 1% of the remaining principal. Based on the above assumptions, the OAS of the new mortgage turns out to be −36.0 basis points. Using this result, we are ready to calculate the refinancing efficiency using formula (2).

Table 1. USD swap curve of 12 November 2004

Term	2 Years	5 Years	7 Years	10 Years	15 Years	30 Years
Rate (%)	3.20	3.97	4.32	4.67	5.06	5.33

The (cash flow) present value of the new 5.50% mortgage is 108.628% and its option value is 8.628%. The present value of the outstanding 5.75% mortgage is 111.640% and its option value is 10.874%. Assuming that the transaction cost is 1%, refinancing would save $111.640 - (108.628 + 1.00) = 2.012\%$ and the reduction in option value would be $10.874 - 8.628 = 2.246\%$. The resulting efficiency would be $2.012/2.246 = 89.6\%$. Because refinancing is not recommended at efficiency below 95%, we conclude that a 30-year 5.75% mortgage should *not* be refinanced with a like mortgage whose rate is 5.50% and transaction cost is 1%.

What if the rate of the outstanding mortgage happened to be higher than 5.75%? Figure 1 compares the savings to the change in option value for a range of rates on outstanding mortgages and Fig. 2 shows the refinancing efficiency. The 100% efficiency threshold is reached when the rate on the outstanding mortgage is 5.94%. We conclude that if the transaction cost is 1% of the remaining principal, a long-term mortgage should be refinanced only if its rate exceeds the current market rate by at least 44 basis points.

The above analysis can be repeated for different transaction costs. Obviously the higher the transaction cost, the wider will be the refinancing threshold. In particular, at a 2% transaction cost the decline in rates required for 95% and 100% efficiency are 53 basis points and 70 basis points, respectively.

III. The Effect of Interest Rate Volatility

Refinancing efficiency obviously increases if either the rate of the new mortgage or the transaction cost decline, with all other factors held constant. But how does interest rate volatility affect efficiency?

When the optionless yield curve is known, volatility affects only option values and not savings; the higher the volatility the greater the option value. Therefore if the new loan happened to be noncallable, higher volatility would reduce the efficiency.

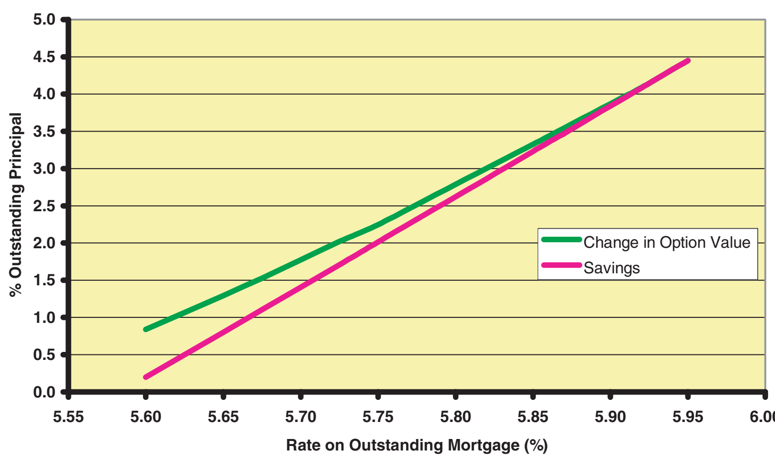

Fig. 1. 30-Year mortgages refinanced with 30-year 5.50% mortgage (Borrower's horizon = 30 years)

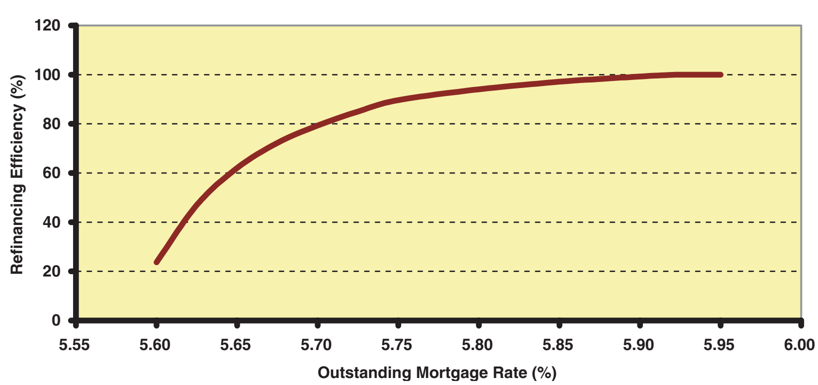

Fig. 2. Refinancing efficiency of 30-year mortgages refinanced with 30-year 5.50% mortgage (Borrower's horizon = 30 years)

But the situation at hand is considerably more complicated. As the new mortgage also has an option, the OAS that calibrates the new mortgage is volatility dependent. Therefore both the savings and the difference between the old and new option values are affected by the choice of volatility.

Before looking at examples, let us consider the relevant range for volatility. The volatility in question is that of the short-term rate. The closest matching market-implied volatilities are those of short-dated American swaptions and they have been generally between 12% and 30%, depending on the shape and level of the yield curve.

Figure 3 displays the effect of volatility on the efficiency of refinancing a 5.75% mortgage and a 5.94% mortgage with a 5.50% mortgage. Recall that at 16% volatility the 5.94% mortgage should be refinanced while the 5.75% mortgage should not.

First, observe in Fig. 3 that the refinancing efficiency of both mortgages declines as volatility increases. Since within the relevant range the efficiency of the 5.75% mortgage is well below 100%, we can conclude that a 25 basis point savings in coupon is insufficient. In contrast, the efficiency of the 5.94% mortgage exceeds 98% even at 30% volatility.

Figure 3 displays another interesting property of refinancing efficiency. As volatility increases, instead of steadily declining, efficiency actually approaches an *asymptote*. This asymptote is roughly 85% for the 5.75% mortgage and 96% for the 5.94% mortgage. The existence of an asymptote has an important practical ramification: if the efficiency is near 100% even at a *high volatility*, there is clearly little justification for waiting.

In practice, 30% is certainly a reasonable upper bound for interest rate volatility. Figure 4 shows the efficiency of refinancing various mortgages at this volatility, keeping the coupon of the new mortgage at 5.50%. The efficiency reaches 100% just below 6.05%, signaling that any mortgage with a higher coupon rate should be refinanced immediately. This leads us to conclude that if the transaction cost is 1% of the remaining principal, annual savings exceed 55 basis points (6.05%–5.50%) warrant the refinancing of

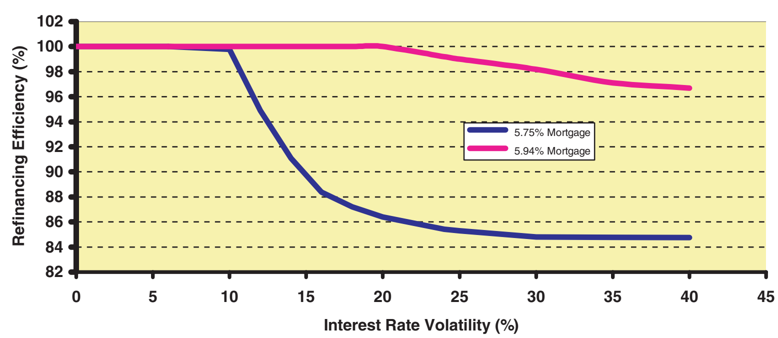

Fig. 3. Refinancing efficiency declines and levels off as volatility rises for 30-year mortgages refinanced with 5.50% mortgage (Borrower's horizon = 30 years)

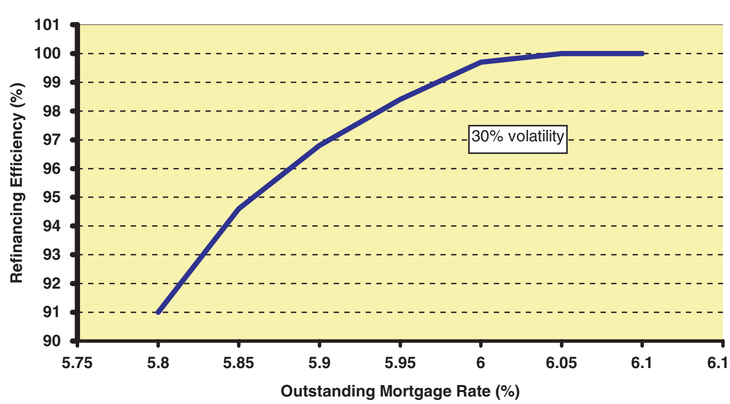

Fig. 4. At 55 bps savings, refinancing is a must 30-year mortgages refinanced with 5.50% mortgage (Borrower's horizon = 30 years)

a long-term mortgage even under the most aggressive volatility assumptions!

But there is a caveat: until now we have assumed that the homeowner's borrowing horizon coincides with the maturity of the outstanding mortgage – 30 years in our illustration. Otherwise the analysis has to be modified in the manner discussed below.

IV. Borrower-Specific Considerations: Horizon and Taxes

In the previous section we assumed that the homeowner's borrowing horizon coincided with the maturity of the outstanding mortgage. But what happens if the horizon is shorter? This arises, for example, if the homeowner expects to be relocated within a specified number of years. Personal income taxes add an additional degree of complexity to the decision. In this section we will show to incorporate such borrow-specific factors into the analysis. In other words, the refinancing efficiency calculation can be customized to fit the parameters of individual homeowners.

Assume that the homeowner's borrowing horizon is only five years. Common sense tells us that a 30-year fixed coupon mortgage is not the proper refinancing (or financing) vehicle for this homeowner, because the cost of a mortgage is increased by the presence of an embedded option that, from the homeowner's perspective, is worthless beyond five years. The right choice is a 30-year adjustable rate mortgage (ARM): the coupon is fixed for five years and is reset annually afterwards (a so-called 5/1 ARM). For a homeowner whose horizon is 5 years, exposure to floating rates beyond the fifth year is irrelevant. Because the rate of a 30-year fixed coupon mortgage is always higher than the rate of a 30-year 5/1 ARM, a rational homeowner will not even consider the former.

From the perspective of this homeowner, a 30-year mortgage amortizes over 30 years but has a 5-year balloon payment. With this insight, we can calibrate the interest rate model to the new 30-year 5/1 ARM rate, which in our example is 5.00%.

The PV of the cash flows for a new 30-year 5/1 ARM turns out to be 100.416% and the value of its call option is 0.416%. Figure 5 shows the savings and the change in option value that would result from refinancing mortgages with various coupons and Fig. 6 shows the resulting efficiencies. The 100% threshold is reached when the rate on the outstanding mortgage is 5.48%, 48 basis points above the current 5% rate.

Finally, we observe that personal income taxes can also be incorporated into the refinancing efficiency formula. The most obvious effect is that mortgage interest payments and up-front points are tax-deductible, however transaction costs are not. But taxes also enter into the discounting process and into option valuation. Consider the example at the beginning of Section I (5.75% mortgage refinanced with 5.50% mortgage). Keep all inputs the same as before and in addition assume that the borrower's marginal income tax rate (combined federal, state and local) is 35%. First, we calibrate the interest rate process so that a 5.50% mortgage is fair on an after-tax basis; the resulting OAS is −71 basis points. Then the after-tax PV of the outstanding mortgage is 111.025%, that of the new mortgage is 108.687% and the corresponding after-tax option values are

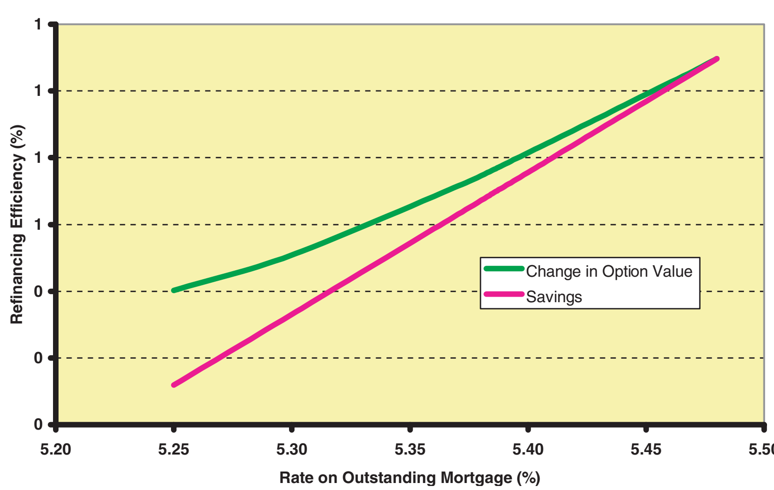

Fig. 5. 30-Year mortages refinanced with 30-year 5.00% ARM fixed for 5 years (Borrower's horizon = 5 years)

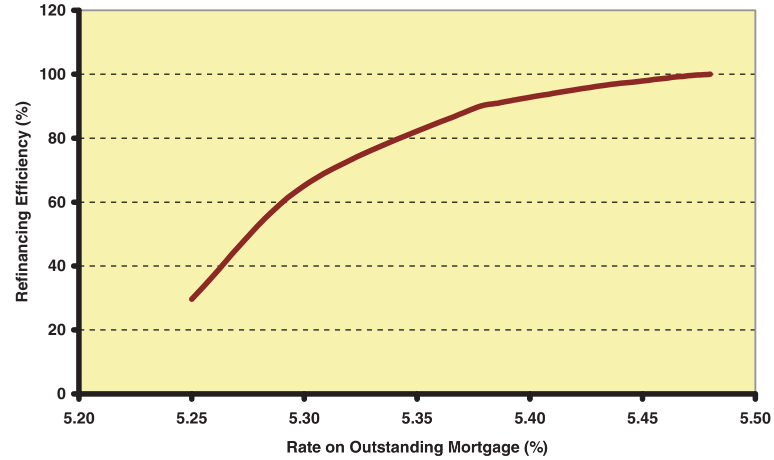

Fig. 6. Refinancing efficiency of 30-year mortages refinanced with 30-year 5.00% ARM fixed for 5 years (Borrower's horizon = 5 years)

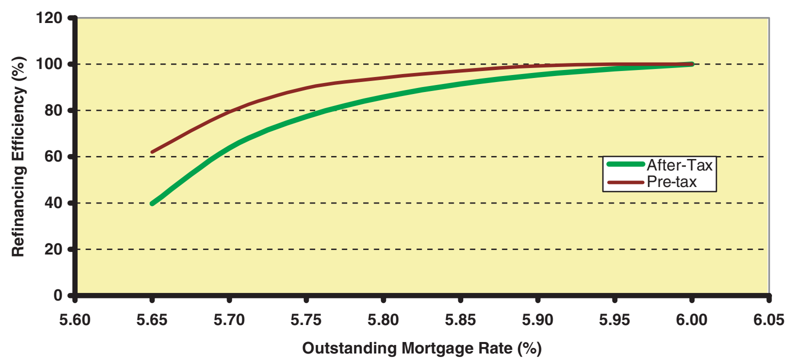

Fig. 7. After-tax refinancing efficiency of 30-year mortgages refinanced with 30-year 5.50% mortgage (Borrower's horizon = 30 Years)

10.417% and 8.687%, respectively. Thus the savings amount to 1.338%, the net change in after-tax option value is 1.730% and the after-tax refinancing efficiency is approximately 77.3%, significantly lower than in the pre-tax case. The intuition behind this result is that transaction costs are not tax-deductible. The payback period measured in annual after-tax savings is longer than it would be in the absence of taxes.

Figure 7 overlays after-tax refinancing efficiency on the pre-tax efficiency shown in Fig. 2. In the after-tax analysis, the 100% threshold is reached when the rate of the outstanding mortgage is roughly 6% (i.e. 50 basis points above the prevailing 5.50% rate). In the absence of taxes, the 100% threshold is reached with an outstanding mortgage whose coupon is 5.94%.

V. Summary and Conclusions

We have shown how to employ contemporary bond analytics to decide whether or not to refinance a conventional residential mortgage. The source of the problem is that refinancing entails transaction costs and the combined cost of repeated refinancings is significant. But not capturing available savings is risky because a sudden rate increase will reduce or completely eliminate these savings. Therefore any realistic solution to the refinancing decision must take into account the optionality of both the outstanding and the new mortgage.

We have shown how the notion of refunding efficiency can be adapted to determine the optimum time to refinance. The basic inputs into the analysis are the terms of the outstanding and the new mortgage and the transaction cost. As an optionless mortgage yield curve is unavailable, an appropriate benchmark yield curve needs to be calibrated to the new mortgage. The calculation can be customized to homeowner-specific parameters, namely borrowing horizon and personal income taxes. The approach can be applied in a straightforward manner to determine the refinancing efficiency under any reasonable set of assumptions.

We found that, among other factors, the refinancing threshold is very sensitive to the transaction cost. At a 1% transaction cost, a long-term fixed coupon mortgage should be refinanced when interest rates have declined roughly 45 basis points, at a 2% cost the threshold increases to about 70 basis points. These findings are remarkably consistent with the 0.5% rule of thumb currently in vogue.

References

Archer, W. R. and Ling, D. C. (1993) Pricing mortgage-backed securities: integrating optimal call and empirical models of prepayment, *Real Estate Economics*, **21**, 373–404.

Archer, W. R. and Ling, D. C. (1997) Demographic versus option-driven mortgage terminations, *Journal of Housing Economics*, **6**, 137–63.

Bennett, P., Peach, R. and Peristiani, S. (2000) Implied mortgage refinancing thresholds. Research and Market Analysis Group, Federal Reserve Bank of New York.

Boyce, W. M. and Kalotay, A. J. (1979a) Optimum bond calling and refunding, *Interfaces*, **9**, 36–49.

Boyce, W. and Kalotay, A. (1979b) Tax differentials and callable bonds, *Journal of Finance*, **34**, 825–38.

Campbell, J. Y. and Cocco, J. F. (2003) Household risk management and optimal mortgage choice, *The Quarterly Journal of Economics*, **118**, 1449–94.

Chinloy, P. (1991) The option structure of a mortgage contract, *Journal of Housing Research*, **2**, 21–38.

Chen, A. H. and Ling, J. C. (1989) Optimal mortgage refinancing with stochastic interest rates, *AREUEA Journal*, **17**, 278–99.

Deng, Y. (1997) Mortgage termination: an empirical hazard model with stochastic term structure, *Journal of Real Estate Finance and Economics*, **14**, 309–31.

Deng, Y., Quigley, J. M. and Van Order, R. (2000) Mortgage termination, heterogeneity and the exercise of mortgage options, *Econometrica*, **68**, 275–308.

Dunn, K. B. and McConnell, J. J. (1981a) A comparison of alternative models for pricing GNMA mortgage-backed securities, *Journal of Finance*, **36**, 471–83.

Fabozzi, F. J. (2006) *The Handbook of Mortgage-Backed Securities*, 6th edn, McGraw Hill, New York, NY.

Finnerty, J. D. and Emery, D. R. (2001) *Debt Management: A Practitioner's Guide*, Harvard Business School Press, Boston, MA.

Hilliard, J., Kau, J. B. and Slawson, C. (1998) Valuing prepayment and default in a fixed-rate mortgage: a bivariate binomial options pricing technique, *Journal of Real Estate Economics*, **26**, 431–68.

Hoover, G. L. (2003) The mortgage refinance decision: an equation-based model, *Financial Services Review*, **12**, 319–37.

Howard, C. D. and Kalotay, A. J. (1988) Embedded call options and refunding efficiency, in *Advances in Futures and Options Research* (Ed.) F. Fabozzi, JAI Press Inc., Greenwich, CT, pp. 97–117.

Kalotay, A. and Tuckman, B. (1999) Subsidized borrowing and the discount rate: the case of municipal capital budgeting and financial management, *Municipal Finance Journal*, **19**, 38–45.

Kalotay, A., Williams, G. and Fabozzi, F. J. (1993) A model for valuing bonds and embedded options, *Financial Analysts Journal*, **49**, 35–46.

Kalotay, A., Yang, D. and Fabozzi, F. J. (2004) An option-theoretic prepayment model for mortgages and mortgage-backed securities, *International Journal of Theoretical and Applied Finance*, **7**, 949–78.

Kalotay, A., Yang, D. and Fabozzi, F. J. (2007) Refunding efficiency: a generalized approach, *Applied Financial Economics Letters*, **3**, 141–6.

Kariya, T. and Kobayashi, M. (2000) Pricing mortgage backed securities (MBS): a model describing the burnout effect, *Asian Pacific Financial Markets*, **7**, 182–204.

Kau, J. B. and Slawson Jr, V. C. (2002) Frictions, heterogeneity, and optimality in mortgage modelling, *Journal of Real Estate Finance and Economics*, **24**, 239–60.

Kau, J. B., Keenan, D. C., Muller III, W. J. and Epperson, J. F. (1992) A generalized valuation model for fixed-rate residential mortgages, *Journal of Money, Credit and Banking*, **24**, 279–99.

Kau, J. B., Keenan, D. C., Muller, W. J. and Epperson, J. F. (1995) The valuation at origination of fixed rate mortgages with default and payment, *Journal of Real Estate Finance and Economics*, **11**, 5–36.

Krainer, J. and Marquis, M. (2003) Mortgages as recursive contracts. Federal Reserve Bank of San Francisco, Working Paper 2003–03.

LaCour-Little, M. (1999) Another look at the role of borrower characteristics in predicting mortgage prepayments, *Journal of Housing Research*, **10**, 45–60.

LeRoy, S. F. (1996) Mortgage valuation under optimal prepayment, *Review of Financial Studies*, **9**, 817–44.

Longstaff, F. A. (2002) Optimal refinancing and the valuation of mortgage-backed securities. Working Paper, University of California at Los Angeles.

Pliska, S. R. (2005) Mortgage valuation and optimal refinancing. Working Paper, Department of Finance, University of Illinois at Chicago.

Roncoroni, A. and Moro, A. (2005) Flexible rate mortgages. Working Paper, ESSEC.

Stanton, R. (1995) Rational prepayment and the valuation of mortgage-backed securities, *Review of Financial Studies*, **8**, 677–708.

Stanton, R. and Wallace, N. (1998) Mortgage choice: what's the point? *Real Estate Economics*, **26**, 173–205.