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Tax Differentials and Callable Bonds

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I. Introduction

A NUMBER OF INVESTIGATORS have considered reasons why various options are included when bonds are issued. Most attention centers on the almost invariable inclusion in corporate bonds of a call or redemption option that enables the borrowing corporation to buy back the bond at a stated price prior to maturity. Such an option provides the borrower with considerable financial flexibility[1] particularly the opportunity to replace the bonds with a lower-cost issue should rates fall.[2]

To obtain this flexibility, the borrower must pay an interest rate premium. While corporations have almost always been willing to pay this premium, the Federal government has not.[3] Since calls are primarily exercised to take advantage of lower interest rates, most arguments for the existence of the call options have emphasized differences between the borrowers' and lenders' predictions and attitudes about interest rate changes (see [8]). But although these might justify the existence of some call options, they do not explain why virtually all the corporate bonds would have this feature, or conversely why "puttable" bonds, which the holder could present for redemption at his option at pre-set price, would be so rare.

The published justifications for the existence of call options are so unconvincing that many authorities have held that there are no valid reasons, in particular that in an efficient market the interest rate premium demanded for the call

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[1] By calling the bonds the borrower can simplify his financial structure, reduce his debt-equity ratio, eliminate indenture restrictions, or lengthen his average debt maturity. Also, if an asset financed with the debt is sold, or destroyed and the loss compensated by insurance, the proceeds can be used to erase the debt.

[2] This use of the call option, called refunding, is so important that lenders sometimes expressly prohibit it even when a simple redemption using cash, equity funds, or higher-cost debt is allowed. (Refinancing refers to the replacement of a mature debt issue by a new borrowing.)

[3] As Pye observed in 1966 [4], "An interesting fact to explain is why most corporate bonds are callable and most public bonds are not."

option would negate the benefits.[4] Nevertheless, almost all corporate bonds are callable, and the prevalence of the custom indicates that some fundamental force is at work, not just haphazard differences of opinion.

In this paper we argue that a compelling reason for the prevalence of callable corporate bonds, and for the rarity of noncallable or puttable corporate bonds, is a particular tax effect, which results from the difference in marginal tax rate between a profitable corporate borrower and the typical lender. Specifically, for a simple but significant model of option provisions and interest rate changes and two related evaluation methods, we demonstrate that when the borrower has a higher tax rate than the lender, a callable bond will be preferred over a noncallable one, while conversely, when the borrower has a lower tax rate than the lender, a puttable bond will be preferred. Since profitable corporations have marginal tax rates of about 50%, while the dominant lenders have marginal tax rates which are significantly lower or even zero (i.e., pension funds), we conclude from our models that corporations have an incentive to borrow using callable bonds.

The analogy between callable bonds and puttable bonds was surprising, and we did not realize at first that there are widely-held puttable bonds which fit the theory, namely U. S. Savings Bonds. Both E and H bonds will be redeemed by the government at the lender's option. Since the government pays no income taxes and most purchasers of the bonds have a positive marginal tax rate, the tax effect makes this type of bond advantageous to both borrower and lender.

A brief outline of our argument is as follows: when transactions are stated after taxes, it is clear that those having higher marginal tax rates have lower after-tax discount rates, and low-tax parties have high discount rates. High- and low-tax parties will value a level stream of interest payments equally, but if the stream is uneven, e.g., increasing or decreasing, those with lower taxes will have a relatively greater preference for receiving decreasing streams and paying increasing streams, due to their higher after-tax discount rate. A high-tax borrower and a lower-tax lender can then obtain mutual benefits from an interest schedule which is declining, and mutual expected benefits from an expected interest schedule which is declining. Since the interest payments on a callable bond and its refunding issue can decline but never increase, a callable bond provides precisely the type of expected interest schedule that a corporate borrower and tax-exempt lender would prefer.

Scholes [15] has observed that taxes also affect other types of options, but that effect is distinct from ours since it is based on the special tax treatment of capital gains. Also, he found that those with high taxes should sell stock options to those with lower marginal tax rates, while we find that low-tax lenders should sell bond options (i.e., buy bonds despite the retention of the call option) to high-tax borrowers.

II. Taxes and the Discount Rate

Consider now a bond of unit principal paying R per year for M years. On a pre-tax basis the lender's flows form the sequence $\{-1, R, \ldots, R+1\}$ ($M+1$ terms).

[4] A notable statement of this position was given by Myers [7], who concluded: "... either that bond markets are not efficient, or that I am wrong in presuming that efficient markets imply that it is not worth trying to reduce interest costs by aggressive use of the call option." A more recent, unhedged example is due to Kraus [9]: "... a policy of issuing only noncallable bonds is as good as that of issuing

If the lender's discount rate is r', then his present value is nonnegative and the bond acceptable if and only if $r' \leqslant R$, so that R is the "internal rate of return" (IRR) of the sequence. For the issuer of the bonds, facing the negative of the lender's flows, the opposite relation holds. The standard conclusion is then that when there is competition among lenders and borrowers, resulting in a prevailing coupon rate of R, then the appropriate pre-tax discount rate for the marginal borrower and lender is R.

However, when taxes are considered, we see that the taxable lender receives not the stream $\{-1, R, R, \ldots, R + 1\}$, but instead $\{-1, uR, uR, \ldots, uR + 1\}$, where $u = 1 - \tau$ and τ is his marginal tax rate, so that u is his after-tax or retention rate. This stream is then acceptable if and only if his after-tax discount rate r' satisfies $r' \leq uR$, so that in analogy to the pre-tax case, the appropriate discount rate for after-tax flows for the marginal lender with tax rate τ and after-tax rate $u = 1 - \tau$, when the coupon is R, is $r' = uR$. For the borrower similar conditions hold, with his after-tax discount rate r'' equal to $u'R$, where u' is his own after-tax rate.

We note that for the particular "level" cash flow analyzed, the after-tax discount rate uR is simply the "after-tax rate" u times the pre-tax discount rate R, which is equal to the coupon on the bond. Because of the importance of this special case, it is useful to express an after-tax discount rate r' in the form $r' = ur$, where u is the after-tax rate of the party, and r is close to (perhaps even equals) the pre-tax discount rate. In the remainder of this paper we adopt this convention, so that when we refer to a party with $u = 0.6$ using a discount rate of 8%, we really mean that he is using an after-tax discount rate of 4.8%.

In the foregoing we have observed the elementary rule that it is the after-tax flows that are significant, and that when discounting such after-tax flows one must use an after-tax discount rate, not a pre-tax one. An early failure to observe this rule marred the first published analysis of the call option for bonds, that of Crockett [2]. She analyzed the flows to the lender and borrower both before and after taxes, but used the same discount rate in both cases. If she had used the proper after-tax discount rate in the after-tax case, the effect we describe in this paper would have been recognized from the outset.

III. Assumptions

We use a simple model to clarify the effects of the differing tax rates. There are only two periods, each of length M, with M long enough (e.g., 15 to 20 years) that the term differential between interest rates of bonds of maturity M and $2M$ is negligible. All parties are risk-neutral, transaction costs are zero, and during each period interest is constant and paid continuously. Non-callable one-period bonds ("M-bonds") are priced competitively, and we shall investigate the feasibility of various types of bonds of maturity $2M$. We let r_1 denote the interest rate on M-bonds during the first period ($t = 0$ to $t = M$), ρ the interest on M-bonds during the second period ($t = M$ to $t = 2M$), and r_2 the expected value of ρ, $r_2 = E[\rho]$.

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callable bonds and subsequently following an optimal refunding policy." Earlier, Weingartner [5] had concluded that since "calling involves incurring an avoidable cost," issuing callable bonds was futile unless one wished to engage in "interest rate speculation."

In considering the various types of $2M$-bonds we will examine both the internal rate-of-return (IRR) of the expected flows, and also the expected present values when the flows are discounted at the current M-bond rate. We obtain essentially the same conclusions from both methods, but since the analysis is much simpler using internal rates of return, we shall concentrate on that case. Analysis of the expected present value under current (random) discount rates has been done only numerically and will be described in a later section.

For the IRR analysis, we first consider a bond with unit principal issued at 0 with maturity at $2M$ which pays interest at a pre-tax rate of r_1 from 0 to M and ρ from M to $2M$. When the after-tax or retention rate is $u = 1 - \tau$, then the post-tax interest is ur_1 from 0 to M and $u\rho$ from M to $2M$. The present value of this bond at issuance using a discount rate of ur is denoted $W(u, r, r_1, \rho)$, which may be expressed as

$$\begin{aligned} W(u, r, r_1, \rho) &= -1 + \int_0^M e^{-urt} ur_1 \, dt + \int_M^{2M} e^{-urt} u\rho \, dt + e^{-2urM} \\ &= -1 + \left(\frac{r_1}{r}\right)(1 - e^{-urM}) + e^{-urM}\left(\frac{\rho}{r}\right)(1 - e^{-urM}) + e^{-2urM} \\ &= \left(\frac{1}{r}\right)(1 - e^{-urM})(-r(1 + e^{-urM}) + r_1 + \rho e^{-urM}). \end{aligned}$$

Since W is linear in ρ, the expected value of $W(u, r, r_1, \rho)$ is just $W(u, r, r_1, r_2)$. Under the IRR of expected flows approach we set $W(u, r, r_1, r_2) = 0$ and try to determine how r behaves as a function of r_1, r_2, and especially u, since taxes are our main concern. We find that r, r_1, r_2, and u (and M, which is taken to be constant) must satisfy:

$$\frac{r - r_2}{r_1 - r} = e^{urM}.$$

Call the right-hand-side $f(u, r)$ (suppressing M) and the left-hand-side $g(r_1, r_2, r)$. In Figure 1 we plot f and g as functions of r, and we see that the appropriate "pre-tax" IRR r (which we denote $r = G(u)$) is determined by the intersection of the graphs of f and g. It is easy to see that when $u > 0$, the solution $r = G(u)$ will be closer to r_1 than r_2.

It is clear that as u (the after-tax rate) increases, the graph of $f(u, r)$ gets higher and higher. Thus when $r_2 < r_1$, as in Figure 1, higher values of u (that is, lower tax rates τ) are associated with higher values of the IRR r. When instead $r_2 > r_1$, as in Figure 2, higher values of u are associated with lower values of r. Thus $dr/du = dG/du$ always has the same sign as $r_1 - r_2$.

This relationship between r and u has implications for the feasibility of noncallable $2M$-bonds. When alternative investments are rated by IRR r over a $2M$ horizon, with after-tax rate u, a noncallable $2M$-bond with coupon rate R will dominate a pair of M-bonds if $R > G(u)$ and one is a lender, or if $R < G(u)$ and one is a borrower. A borrower with $u = u_b$ and a lender with after-tax rate u_l can agree on a $2M$-bond with coupon R only if $G(u_l) \le R \le G(u_b)$, so necessarily

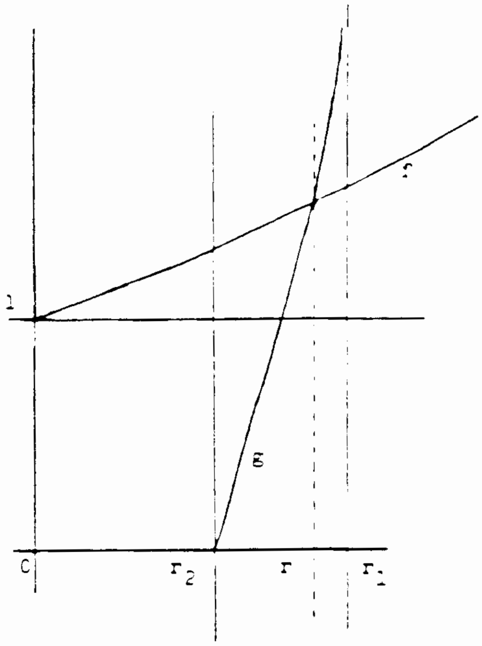

Figure 1

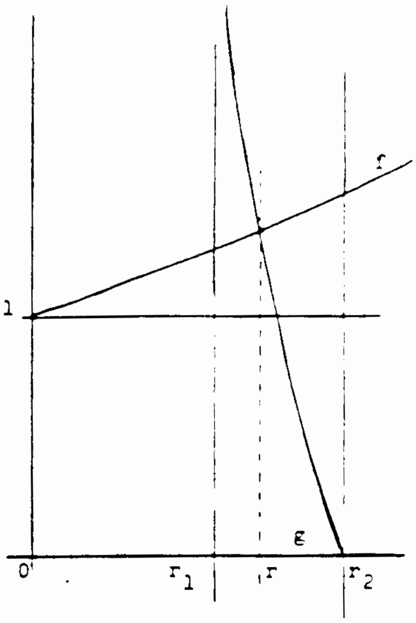

Figure 2

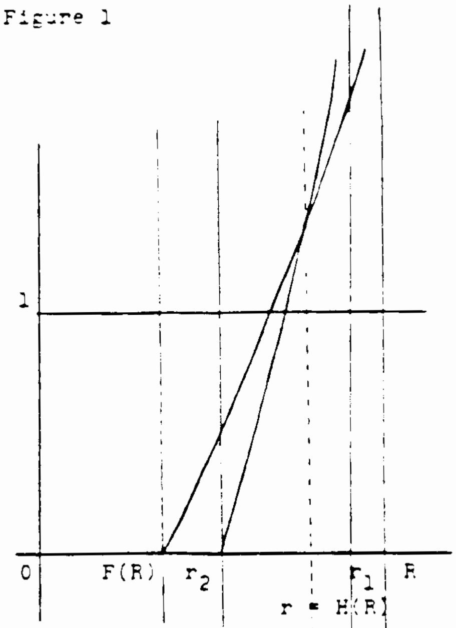

Figure 3

$G(u_l) \leq G(u_b)$. But then either (1) $r_1 \geq r_2$ and $u_b \geq u_l$; or (2) $r_1 \leq r_2$ and $u_b \leq u_l$. Thus if rates are expected to decline, one can borrow $2M$ most advantageously from a lender who has a higher tax rate (lower u), while if rates are expected to increase, one should borrow $2M$ from those with lower tax rates.

IV. Examples

We take $M = 20$ (years), $r_1 = 8\%$, and $E[\rho] = r_2 = 9\%$, 8%, or 7%. Our after-tax rates are $u = 0.5$, corresponding to the corporate tax rate of about 50%, and $u = 1.0$, for either a tax-free institution or the government. Our results appear in the "noncallable" column of Table 1.

When $r_2 = 9\%$, and when $u = 0.5$, the IRR for successive M-bonds is 8.30%, and when $u = 1.0$, the IRR is 8.16%. Thus institutions will lend $2M$ and noncallable to corporations at coupons between 8.16% and 8.30%, while government borrowing and corporate lending will stay in the M-bond market.

When $r_2 = 8\%$, then all will be indifferent between dealing either M or $2M$.

When $r_2 = 7\%$, for $u = 0.5$ the IRR is 7.68% and for $u = 1.0$ it is 7.83%; thus corporations will lend $2M$ and noncallable to the government at coupons between 7.68% and 7.83%, but institutions will demand too much from the corporations for noncallable $2M$-bonds, and corporations will borrow and institutions lend in the M-bond market.

V. Conditional Bonds

Let us consider now "conditional bonds" which mature at $2M$ but which pay interest during the second period (from M to $2M$) which depends on ρ, the M-bond interest during that period. Our (greatly simplified) model of a "callable" $2M$ bond with initial coupon R pays at rate R from 0 to M, and at rate R from M to $2M$ if $\rho \geq R$; while if $\rho \leq R$ it pays at ρ from M to $2M$. (This corresponds to "calling" the existing bond at par at time M and replacing it with an M-bond if this reduces interest.) For this bond, the expected rate from M to $2M$ is $F_c(R) = E[\min(R, \rho)]$. Similarly, our "puttable" $2M$ bond with coupon R pays R from 0 to M, pays R from M to $2M$ if $\rho \leq R$, and pays ρ from M to $2M$ if $\rho \geq R$. The expected rate from M to $2M$ is then $F_p(R) = E[\max(R, \rho)]$. For each R, $F_c(R) \leq R$, and $F_c(R) \leq r_2 = E[\rho]$; similarly $F_p(R) \geq R$ and $F_p(R) \geq r_2$.

What is the IRR of the expected return on these conditional bonds when the after-tax rate is u? Our previous M-bond analysis applies if r_1 is replaced by R and r_2 is replaced by $F_c(R)$ (or $F_p(R)$), so the IRR r must satisfy

$$g(R, F(R), r) = f(u, r)$$

where $F(R)$ denotes $F_c(R)$ or $F_p(R)$ as appropriate.

To assess the attractiveness of conditional bonds, we need to know what values of R will give an IRR equal to that obtainable in the M-bond market. When IRR's are equal, then the values of $f(u, r)$ (the right-hand sides of the defining equations) are equal, and so the left-hand sides must be equal also. Thus we want to determine the points of intersection of $g(r_1, r_2, r)$ and $g(R, F(R), r)$, where $F(R) = F_c(R)$ for callable bonds and $F(R) = F_p(R)$ for puttable bonds.

Since only positive r for which g is positive are relevant, let us consider the intersections of $g(r_1, r_2, r)$ with $g(r_1', r_2', r)$ that lie in the first quadrant. Any r for which $g(r_1, r_2, r) = g(r_1', r_2', r) > 0$ must lie simultaneously between r_1 and r_2, and between r_1' and r_2'. Thus if I is the open interval with endpoints r_1 and r_2 and I' the open interval with endpoints r_1' and r_2', for a point of intersection with $g > 0$ to exist, I and I' must intersect. It is easily verified that if $r_1 - r_2$ and $r_1' - r_2'$ have different signs, then intersection of I and I' is also sufficient for intersection of g and g'. But for the case in which $r_1 - r_2$ and $r_1' - r_2'$ have the same sign, applying the requirement that r be between r_1 and r_2 to the expression

$$r = \frac{r_2 r_1' - r_1 r_2'}{r_1' + r_2 - r_1 - r_2'}$$

reveals that one of the intervals I and I' must be a proper subset of the other.

Now suppose that r_1 and the random variable ρ with expected value $r_2 = E[\rho]$ are given. We have observed that for each after-tax rate $u > 0$ there is an IRR $r = G(u)$, between r_1 and r_2, such that $W(u, r, r_1, r_2) = 0$, and we have seen that $dG/du = dr/du$ has the same sign as $r_1 - r_2$. Now let R denote an initial coupon rate and $F(R)$ be either $F_c(R)$ or $F_p(R)$, as defined previously. If for some R, $g(R, F(R), r)$ and $g(r_1, r_2, r)$ intersect at r with $g > 0$ (see Figure 3), we write $r = H(R)$, and we note that since r is necessarily between r_1 and r_2, the range of H is contained in the range of G. Thus for each u such that $G(u) = H(R)$ for some R, we can define R as a function of u by $R = H^{-1}G(u) = J(u)$. The function J tells us what initial coupon $R = J(u)$ will enable a party with after-tax rate u to be indifferent, on an IRR basis, between successive M-bonds with expected returns r_1 and $r_2 = E[\rho]$ and a $2M$-option bond with expected returns R and $F(R)$. (We note that J depends on r_1 and the distribution of ρ, as well as whether $F(R)$ denotes $F_c(R)$ or $F_p(R)$.) In the Appendix we show that for our callable bond, $dJ/du < 0$, regardless of the sign of $r_1 - r_2$; and that for our puttable bond, $dJ/du > 0$. This implies that for a callable bond, the higher the after-tax rate, the lower the coupon R that yields an IRR equal to that obtainable with M-bonds. Therefore if a lender has an after-tax rate u_l and a borrower an after-tax rate u_b, and $u_l > u_b$, then $J(u_l) < J(u_b)$, so any coupon R between $J(u_l)$ and $J(u_b)$ will be mutually advantageous to borrower and lender. On the other hand, if $u_l < u_b$, no initial coupon R can satisfy both in comparison to the IRR they could obtain in the M-bond market. Thus callable bonds are attractive when the lender has a higher u (lower tax rate τ) than the borrower. In an analogous manner, since $dJ/du > 0$ for puttable bonds, low-u lenders and high-u borrowers can agree on a mutually advantageous coupon on a puttable bond, but high-u lenders and low-u borrowers cannot. Thus for either classification of borrowers and lenders by comparative tax rates, there is an appropriate, mutually advantageous conditional bond.

VI. More Examples

Let us again consider the cases r_2 = 9%, 8%, and 7% under callable bonds, where we take M = 20 years and ρ to be normally distributed, with σ = 2%. (Our results for σ = 2% as well as 1% and certainty appear in Table 1.) If r_1 = 8% and r_2 = 9%, what will be the callable bond coupons which match the IRR for u = 0.5 and u = 1.0? For u = 0.5, we had an IRR of 8.30% in the M-bond market, and we find that this can be matched by a callable bond with an initial coupon of 8.47%, a 17-basis-point premium over the noncallable. The institutional lender with a M-bond IRR of 8.16%, can match this with a callable bond with initial coupon 8.24%, for an 8-basis-point premium. The spread between the two callable bond coupons is 23 basis points. If the bond were sold with the higher 8.47% coupon, the institution would have a present-value gain of 2.58% of the face value of the bond;

Table 1

Internal Rate of Return

	noncallable		callable		puttable	
	u = .5	u = 1	u = .5	u = 1	u = .5	u = 1
r_2 = 9%						
2%	8.30	8.16	8.47	8.24	7.85	7.93
σ 1%	8.30	8.16	8.35	8.18	7.97	7.98
0%	8.30	8.16	8.30	8.16	8.00	8.00
r_2 = 8%						
2%	8.00	8.00	8.29	8.15	7.71	7.86
σ 1%	8.00	8.00	8.15	8.07	7.85	7.93
0%	8.00	8.00	8.00	8.00	8.00	8.00
r_2 = 7%						
2%	7.68	7.83	8.16	8.08	7.50	7.75
σ 1%	7.68	7.83	8.04	8.02	7.63	7.81
0%	7.68	7.83	8.00	8.00	7.68	7.83

while if it were sold at the institutional break-even of 8.24%, the corporation would have a present value gain of 2.01% of the face value. Between 8.24% and 8.47%, commensurate mutual gains are achievable. (This should be compared with the typical issuing costs for new bonds of 1% or less.)

Similarly, when $r_1 = r_2 = 8\%$, corporations would be willing to issue callable bonds with a coupon of 8.29%, while institutions would buy bonds with a coupon of 8.15%, a spread of 14 basis points. If the bonds were issued at the higher rate, the institutions would gain 1.61%; if issued at the lower, the corporations would gain 1.23% of the face value.

When $r_2 = 7\%$, corporations could not profitably borrow $2M$ from institutions using noncallable bonds, but corporations could achieve an IRR of 7.68% with a 8.16%-coupon callable $2M$-bond, while institutions would demand a coupon of only 8.08% to make their IRR of 7.83%. The spread between coupons is reduced to only 8 basis points. At the higher rate, the institution's gain would be 0.88%; at the lower, the corporation's gain is 0.65%.

Note that if r_2 greatly exceeds r_1, the chance of calling becomes small and the callable bond is very close to being noncallable; but in this case, the borrowing corporation and lending institution could agree on a noncallable bond, so the callable bond is still feasible. On the other hand, when r_2 is much less than r_1 the call is almost sure to be exercised, so the callable bond is essentially a pair of M-bonds, which is what was preferred by a borrowing corporation and lending institution in that situation. But it is when σ is large compared to the difference between r_1 and r_2 that the callable bond has truly unique characteristics, providing mutual values to lender and borrower that would not be otherwise available. One might even say that the callable bond *becomes* the type of arrangement that the parties would have preferred if they had known ρ in advance.

Similar considerations apply to puttable bonds. (See Table 1.) Here taxable lenders will always prefer government bonds which have maturity $2M$ and are puttable, regardless of the expected course of interest rates.

VII. The Random Discount Rate Method

We now show that when we consider expected present values computed using current (random) discount rates, the results are remarkably similar to those of the IRR analysis. Let $I(R, \rho)$ denote the interest paid on a bond of maturity $2M$ during the second period, whether the bond is "conditional" or not. For a noncallable bond, $I(R, \rho) = R$, for a callable bond, $I(R, \rho) = \min[R, \rho]$, and for a puttable bond $I(R, \rho) = \max[R, \rho]$. The present value $V(u, r_1, R, I)$ of a $2M$-bond is a random variable which depends on ρ. For a specific realization of ρ, V takes the form

$$V(u, r_1, R, i(R, \rho)) = -1 + \int_0^M e^{-ur_1 t} uR\,dt + e^{-ur_1 M} \cdot \left[\int_0^M e^{-u\rho t} uI(R, \rho)\,dt + e^{-u\rho M}(1)\right]$$

$$= -1 + \left(\frac{R}{r_1}\right)(1 - e^{-ur_1 M}) + e^{-ur_1 M} \cdot \left[\left(\frac{I(R, \rho)}{\rho}\right)(1 - e^{u\rho M}) + e^{u\rho M}\right].$$

To get the expected value $E[V]$ of V we must evaluate the expected value of the bracketed term, which depends on the distribution of ρ.

If our "$2M$-bond" is really just a pair of M-bonds, so that $R = r_1$ and $I(R, \rho) = \rho$, then the bracketed term is identically 1 and the value of V is 0, regardless of ρ. Thus our standard of comparison is $E[V] = 0$. Since $E[V]$ is written in terms of the lender's cash flows, the lender wants $E[V] > 0$, while the borrower prefers $E[V] < 0$. When ρ is certain, so that $\rho = r_2$, we can solve $E[V] = V = 0$ for R in the noncallable case to obtain an expression which can be shown to have the same sign as $r_1 - r_2$, exactly as in the IRR case. Illustrative values for R are given in Table 2 under "$\sigma = 0$".

When ρ is random, we compute the required R by setting $E[V]$ equal to zero. But in this case, we are unable to get closed-form expressions for R, due to the presence of ρ in the denominator. Thus we investigate numerically the relationships that hold among the coupons required for noncallable $2M$-bonds and for our "callable" and "puttable" bonds when ρ is normally distributed, with a standard deviation σ small enough so that the question of "negative ρ" can be neglected. Our results show that there is very little deviation from the $\sigma = 0$ case.

Specifically, we shall once again consider the case where $r_1 = 8\%$ and $r_2 = E[\rho] = 9\%$, 8%, and 7%. We shall assume that the random variable ρ is normally distributed about $r_2 = E[\rho]$ with standard deviations $\sigma = 0\%$, 1%, or 2%. The numerical results are given in Table 2, for noncallable, callable, and puttable bonds, where the values shown are those initial coupons R which give an expected present value $E[V] = 0$.

We find that the coupons for which $E[V] = 0$ are almost exactly the same as those that match IRR's in our simpler method of analysis, and the spreads in

Table 2

Random Discount Rates

		noncallable		callable		puttable	
		u = .5	u = 1.	u = .5	u = 1.	u = .5	u = 1.
r = 9%							
	2%	8.25	8.12	8.44	8.21	7.83	7.91
σ	1%	8.29	8.15	8.34	8.17	7.97	7.98
	0%	8.30	8.16	8.30	8.16	8.00	8.00
r = 8%							
	2%	7.95	7.95	8.27	8.12	7.68	7.82
σ	1%	7.99	7.99	8.14	8.07	7.85	7.92
	0%	8.00	8.00	8.00	8.00	8.00	8.00
r = 7%							
	2%	7.62	7.76	8.14	8.07	7.46	7.69
σ	1%	7.67	7.81	8.03	8.02	7.62	7.79
	0%	7.68	7.82	8.00	8.00	7.68	7.82

particular are very close. Thus the latter method, which is theoretically the more satisfactory, provides essentially the same results as the IRR approach, at least in the cases we have examined.

VIII. Commentary

What are we to conclude about the type of bond which is preferred when borrower and lender are in different tax brackets? As was outlined in the introduction, our findings indicate that because of differences between after-tax discount rates due to tax differentials, conditional bonds such as callable or puttable bonds are mutually advantageous when interest rates vary with time, even when the borrower and lender's expectations are identical.

Call and put provisions in government financing are discussed by Hess and Winn [1]. Government bonds issued during the Civil War were callable during the last half of their life at par, but the call provision was found to have little value except close to maturity. So Federal bonds were eventually standardized as callable in only the last one-fifth of their lifetime, for maturity management purposes. During the First World War a puttable Treasury bond was first offered and was well-accepted. Hess and Winn do not mention that the First World War also marked the institution of the Federal income tax. From then on, Federal bonds were either noncallable or puttable, except that a call provision for the last few years was sometimes retained. After World War II, the Treasury came under pressure to reinstitute callable bonds, but when one was finally tried in 1960, Hess and Winn report that, "The offering was not favorably received for reasons which are by no means clear." According to our theory, a coupon acceptable to the government on a callable bond would be too low to be attractive to taxable lenders.

Bonds of U. S. governmental bodies other than the Federal government and its agencies are usually referred to as "municipal" bonds. For almost all of these, the interest is not subject to the regular Federal income tax,[5] so both borrower and

[5] Although for many holders the interest may be subject to a "minimum tax."

lender have the same zero tax rate. Since the influence of the tax differential is absent (or greatly diminished), the theory indicates that the existence and terms of options on municipal bonds would be more unpredictable than in the corporate or Federal bond markets, and indeed such seems to be the case. In their study of 964 municipal issues, Hess and Winn [1] did not report any puttable bonds (probably including them, if any, as noncallable ones, since they did not even use the term "puttable"); but two-thirds (626) of the issues studied were found to be non-callable, while one-third (338) were callable, with no standard provisions observed as to call deferments and premiums.

A recent proposal is that municipalities be given a "taxable bond option" or "TBO" so that if the municipality issued bonds which paid non-tax-exempt interest, the Federal government would subsidize some proportion α (about 30%) of the interest on the bonds. Since the subsidy would resemble the deductability of interest for a corporation in an α tax bracket, our analysis indicates that TBO bonds would be most advantageously issued as callable bonds.

We note that Series E U.S. Savings Bonds pay all their interest at maturity or redemption, in addition to being puttable. Our theory indicates that this is indeed a desirable arrangement, since it represents a maximum amount of "shaping". An equally extreme case for corporate bonds would be a bond which paid all its interest immediately upon issuance. But a $1000 bond which paid (e.g.) $900 interest on issuance and none thereafter would not provide a $900 tax deduction in the first year. Instead, the $900 would be treated like a discount and be amortized over the life of the bond. If it were a 30-year bond, there would be a $30 tax deduction each year, resulting in a $15-a-year tax savings at a 50% tax rate. This procedure would still be advantageous to the company, e.g., when long-term rates are 8%, a noncallable, non-interest-bearing 30-year discount bond would be acceptable on an interest basis to tax-exempt institutions with a one-time interest payment at issuance of 91% or more, while for the company any interest less than 98.6% would be profitable.[6] It is apparent from this example that it would take an enormous amount of debt (in face value) to meet a new money requirement in this way. What is not immediately apparent is that once such bonds were issued, the cash flows in later years would be greatly increased so as to reduce the new money needed then. For since no actual interest is being paid in subsequent years, but a tax deduction is available, there is a net *inflow* of tax savings due to the debt, rather than a net outflow of interest.

A referee has suggested that the prepayment privilege on residential mortgages be discussed in the light of our findings. For residential mortgages the borrowers are individuals with marginal tax rates typically around 25–28%, while the lenders are banks or savings institutions which are taxed as corporations. Clearly there is a good rationale for the borrower having a right of redemption (e.g., sale of the property, or destruction by fire), but the apparent difference in marginal tax rates would seem to argue against allowing prepayment for refunding, even if modest penalties were assessed. In fact, due to the privilege allowed residential lending institutions to add portions of their pre-tax profits to their loss reserves tax-free, the marginal tax rates of the lenders, particularly the dominant savings-and-loan associations, appears to be quite close to the 25–28% typical of the borrowers [11]. Thus the theory would indicate that the parties would be indifferent to the

[6] Corresponding to discount bonds issued at 90 or 14 per 1000 respectively.

presence or absence of restrictions on prepayments for refundings.

IX. Notes on the Literature

It is something of a historical mystery why investigators so long overlooked the tax effect, and why later authors so readily adopted the position that taxes must not matter. Clearly the fact that the tax effect is zero for a noncallable bond issued at par exerted a subtle influence, since the deviations from that admittedly special case may have appeared to be modest. But a more powerful factor was that the pre-tax or after-tax question was eclipsed very early by a long controversy over what is the proper rate for discounting *pre-tax* flows. Whether to use the cost-of-capital to the firm, the coupon rate of the refunding issue, a risk-free rate, or some other discount rate seemed much more important than whether to discount pre-tax flows at a pre-tax rate or after-tax flows at an after-tax rate, and indeed, the stakes were larger.[7] But by the time the cost-of-capital position had been argued down, the practice of "ignoring tax effects" was well-established.[8] So later authors generally omit taxes, either declaring their effect to be negligible, or citing their omission in earlier work. But when one tries to analyze and justify the call provision, one is dealing with differences of nearly identical quantities, and factors previously neglected because of their small effect on the gross magnitude are no longer available to affect the differences, leading to the erroneous conclusion that the differences are zero.

The fundamental work of Bowlin [3] recognized the relevance of the tax-deductibility of interest to the proper discount rate, and he used the after-tax discount rate in his own calculations; thus in a time of 4%-to-5% bond coupons, he regarded 3.6% as a profitable rate of return on a bond exchange, and in an example, when the rate on the refunding bond was 4%, he discounted after-tax flows at an after-tax rate of 2%. Yet when he critiqued the calculations of others, which was a major objective of his work, he let their using the "pre-tax rate on pre-tax flows" method pass without critical comment. The ambiguity of the era is well illustrated by the fact that when Bowlin specifies how to compute a "synthetic yield" to account for the after-tax net costs of the refunding issue, he then compares it with the *coupon* of the old issue, not its after-tax cost, to determine the advantage of the refunding.

The confusion between after-tax and pre-tax calculations is further illustrated by an example in the literature [16] in which it is specified that after-tax interest costs be discounted at an after-tax discount rate, but the after-tax cost of the final repayment of principal (which equals the pre-tax cost) be discounted at a pre-tax discount rate, since the principal payment is not tax-deductible. On the other extreme, Spiller [17] did all his calculations as we have recommended, but since he was studying only a break-even refunding coupon rate, and for a single tax rate, he did not observe the effect of the tax differential.

A recent study by Leibowitz, privately published in a pamphlet "The Timing

[7] From the beginning authors frequently hedged their discount rate assumptions by stating that they were mainly analyzing the structure of the refunding decision strategy, and that any discount rate could be used equally well in their formulas.

[8] The custom of ignoring taxes appears to be responsible for a recent debate concerning the refunding of discounted bonds; see [12, 13, 14].

of Corporate Refundings" [10], does give the tax effect a leading, if mysterious role, including an effect on the proper discount rate. In this study Leibowitz makes refunding calculations on both a pre-tax and after-tax basis, and he is apparently impressed by the large and unexpected increase in value that arises from the after-tax case when compared with the pre-tax case. He does not infer, however, that since the pre-tax value represents a loss to the tax-exempt lender, and the after-tax value the gain to the taxable borrower, that the differential justifies the existence of the call provision.

Appendix

We wish to show that for our model of a callable bond $dJ/du < 0$, regardless of the sign of $r_1 - r_2$; and that for our puttable bond, $dJ/du > 0$.

For the proof, we observe that when $g(r_1, r_2, r) = g(R, F(R), r)$, r must satisfy

$$r = \frac{r_2 R - r_1 F(R)}{R + r_2 - r_1 - F(R)}$$

If ρ is a continuous random variable, then $F'(R)$ will exist and be continuous, and whether the bond is callable or puttable, we will have $F'(R) \geq 0$. Letting D denote the denominator and differentiating, we obtain

$$dr/dR = (r_1 - r_2)[(r_1 - R)F'(R) + (F(R) - r_2)]/D^2.$$

Now $dJ/du = dR/du = (dr/du)(dr/dR)^{-1}$; since dr/du has the same sign as $r_1 - r_2$, and dr/dR has $r_1 - r_2$ as a factor, dJ/du will have the same sign as the term in brackets. Thus it remains to show that when $G(u) = H(R)$, the term in brackets is negative when $F(R) = F_c(R)$ and is positive when $F(R) = F_p(R)$. From the observed relationships among $r_2 = E[\rho]$, $F_c(R)$, $F_p(R)$, and R, we only need to verify that $R \geq r_1$ for callable bonds when $r_1 > r_2$, and $R \leq r_1$ for puttable bonds when $r_1 < r_2$. For the first case, callable bonds with $r_1 > r_2$, $r_1 - r_2$ and $R - F_c(R)$ have the same sign, and so for intersection one of the intervals must be a subset of the other. But since $F_c(R) \leq r_2$, the interval (r_2, r_1) must be a subset of $(F_c(R), R)$, and so $r_1 \leq R$, which was to be proved. Similarly, when $r_1 < r_2$ for puttable bonds, since $r_1 - r_2$ and $R - F_p(R)$ are both negative, one of the intervals must be a subset of the other, and $(R, F_p(R))$ must be the larger interval, so that $R \leq r_1$. Thus the crucial point in the proof is that the IRR on the conditional bond must actually equal the IRR on successive M-bonds for some u in order for the function J to be defined.

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