Subsidized Borrowing and the Discount Rate: The Case of Municipal Capital Budgeting and Financial Management

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Congress has allowed municipalities to issue bonds with interest payments that are exempt from federal taxation. As a result, municipalities borrow at lower risk-adjusted interest rates than those available to the U.S. government, its agencies, and private corporations. How does the opportunity to borrow at tax-exempt rates affect the rate at which a municipality should discount future cash flows?

While a correct answer to this question is crucial for both capital budgeting and debt refinancing decisions, existing scholarly articles treat the discount rate as a nuisance parameter—that is, as incidental to their main pursuits. As a result, practitioners seeking guidance must search for clues through articles about underwriting auctions (Walker 1969, West 1968, and West 1969), direct and advance refunding (Dyl and Joehnk 1976), the subsidization of municipal capital (Gordon and Metcalf 1992), and the valuation of embedded options (Gurwitz, Knez, and Wadhwani 1992). Worse yet, these incidental references recommend the use of

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different discount rates.[1] This article focuses on the choice of a discount rate for municipalities and argues that the correct discount rate is, in fact, the taxable rate of interest.

The (incorrect) argument for using the tax-exempt rate can be stated as follows. First, a municipality borrows at the tax-exempt rate. Second, because of arbitrage restrictions imposed by the federal government, a municipality cannot invest at a rate exceeding the tax-exempt rate. It is for this reason, in fact, that the U.S. Treasury sells State and Local Government Series (SLGS) securities, that is, Treasury securities that pay a municipality the maximum rate it can earn without violating arbitrage regulations. It follows from these two statements that a municipality transforms current dollars into future dollars, and vice versa, at the tax-exempt rate. Consequently, the tax-exempt rate is the correct discount rate for capital budgeting and financial management decisions.

The flaw in this argument is that municipalities can and do invest funds at the taxable rate.[2] Federal arbitrage restrictions apply only to funds borrowed at the tax-exempt rate, not to funds raised from other sources—for example, taxes, capital project revenues, and, in rare cases, taxable market borrowings. Furthermore, because state and local laws typically permit tax-exempt borrowing only to finance capital projects, almost all municipalities do use these other sources of funds to finance their spending that cannot be legally financed by tax-exempt borrowing.

From these facts one can argue that the taxable rate, rather than the exempt rate, is the appropriate discount rate. First, municipalities limit their borrowings in the tax-exempt market because there are a limited number of worthwhile capital projects and, sometimes, because there are legal debt ceilings on total borrowings. Therefore, the tax-exempt rate does not necessarily represent the *marginal* opportunity cost of funds. Second, since municipalities carry balances of unrestricted funds and invest these at the taxable rate, the taxable rate does represent the *marginal* opportunity cost of funds.

The following analogy may be useful in understanding this argument.[3] Consider a firm that has an opportunity to borrow up to $100,000 from a government agency at 3 percent while prevailing market rates are

1. West (1968 and 1969) recommends the taxable rate but Walker (1969) disagrees. Dyl and Joehnk (1976) and Gurwitz, Knez, and Wadhwani (1992) use the tax-exempt rate. Finally, Gordon and Metcalf (1992) imply the use of the after-tax taxable rate when projects are financed by taxes and the use of the exempt rate when projects are financed by borrowing.

2. According to Federal Reserve data as of the start of 1992, the financial assets of state and local governments, excluding receivables, totaled $683.9 billion. Of this amount, over 48 percent was definitely invested in taxable securities and slightly over 2 percent was definitely invested in tax-exempt securities. The remaining 50 percent was invested in Treasury issues. Unfortunately, however, the Federal Reserve does not report the amount of SLGS included in the Treasury issue total.

3. A similar example appears in Brealey and Myers (1991), at 288.

7 percent. While the opportunity to borrow at 3 percent has positive net present value, it does not determine the firm's marginal cost of funds. Similarly, a municipality that limits its borrowings at the tax-exempt rate and invests its marginal dollar at the taxable rate must consider the taxable rate as its discount rate.

This article presents a model that formalizes this argument and presents two applications of the analysis—refunding existing debt and choosing between capital projects. These applications illustrate how using the tax-exempt rate to discount cash flows leads to incorrect decisions.

A MODEL OF MUNICIPAL FINANCE

The model captures one year of financing municipal expenditures. At the start of the year, on date 0, the municipality collects tax revenues of T dollars, borrows B dollars through at the tax-exempt rate r_E, and invests K dollars in capital projects. In the middle of the year, on date 1, the municipality pays Q_1 dollars to meet noncapital expenditure requirements, pays interest on its debt of $Br_E/2$, and collects revenue from its capital projects of $R_1(K)$. At the end of the year, on date 2, the municipality pays Q_2 dollars in noncapital expenditures, pays interest and principal on its debt of $B(1 + r_E/2)$, and collects project revenues of $R_2(K)$. Finally, any cash balances or shortfalls carried between dates are invested or financed at the taxable rate $r_T > r_E$.

The objective of the municipality is to choose T, K, and B so as to minimize the tax it imposes on the citizenry subject to the constraint of meeting its noncapital expenditure requirements on dates 1 and 2.[4] Capital spending helps to lower taxes by producing profitable revenue flows.[5] Assume that available capital projects are attractive enough to attract some positive level of municipal spending. Also assume that available capital projects are not so plentiful that they enable the municipality to finance its noncapital expenditures from project revenues alone—that is, without any tax revenues. Finally, assume that increased capital investment provides diminishing marginal returns in each period, that is, $\partial^2 R_i / \partial K^2 < 0$, $i = 1,2$.

The municipality faces two constraints. First, since it must make its noncapital expenditures, it must have no cash shortfall at the end of the

4. If the municipality were not able to borrow or invest in capital projects, its objective would be to meet noncapital expenditure requirements in the least-cost way, that is, by minimizing taxes. The opportunity to borrow and invest in capital projects does not change this objective. Rather, borrowing and investing can also generate revenue and allow the municipality to lower taxes even further.

5. This formulation ignores any externalities that might accrue to the citizenry from a municipality's capital projects. The formulation also assumes that a municipality can choose the level of capital spending. For the special case of choosing between mutually exclusive capital projects, see "Choosing Among Capital Projects."

year. Second, $B \leq K$. This constraint reflects federal arbitrage restrictions that prevent a municipality from borrowing in the municipal market at the rate r_E and investing those very funds in the taxable market at the higher rate r_T. Alternatively, funds borrowed through the tax-exempt market may be spent only on capital projects.

The discussion now turns to the mathematical expression of the problem described above. The municipality's cash balance on date 2 can be easily shown to be

$$[(T + B - K)(1 + r_T/2) - Br_E/2 + R_1(K) - Q_1] \\ (1 + r_T/2) - B(1 + r_E/2) + R_2(K) - Q_2.$$

Therefore, the municipality's minimization problem can be written as follows. $\text{Min}_{T,K,B}\ T$ such that

$$[(T + B - K)(1 + r_T/2) - Br_E/2 + R_1(K) - Q_1] \\ (1 + r_T/2) - B(1 + r_E/2) + R_2(K) - Q_2 \geq 0$$

$$K \geq B$$

Let λ_1 and λ_2 be the Lagrange multipliers for the two constraints of the problem, respectively. In economic terms, these multipliers represent the shadow prices of the constraints, that is, the marginal value of relaxing the constraints.

The first-order conditions of the municipality's problem give rise to the following conditions:

$$\lambda_1 = \frac{1}{\left(1 + \frac{r_T}{2}\right)^2} \tag{1}$$

$$\lambda_2 = \frac{\frac{1}{2}(r_T - r_E)}{\left(1 + \frac{r_T}{2}\right)} + \frac{\frac{1}{2}(r_T - r_E)}{\left(1 + \frac{r_T}{2}\right)^2} \tag{2}$$

$$\frac{R_1'(K) - \frac{r_E}{2}}{\left(\frac{1 + r_T}{2}\right)} + \frac{R_2'(K) - \left(1 + \frac{r_E}{2}\right)}{\left(1 + \frac{r_T}{2}\right)^2} = 0 \tag{3}$$

$$B = K \tag{4}$$

These equations implicitly define how municipalities should discount future cash flows. Recall that λ_1 is the value, in terms of today's tax dollars, of relaxing the year-end, no-deficit constraint. In other words, λ_1

is the present value of \$1 to be received on date 2. Equation (1) states that future dollars should be discounted at the taxable rate of interest.

Recall that λ_2 is the present value of relaxing the constraint on tax-exempt borrowing, that is, the value of borrowing another dollar at the rate r_E. Since borrowing \$1 at the rate r_E allows the municipality to keep an extra \$1 of tax revenue invested at the rate r_T, tax-exempt borrowing provides a boon of $1/2(r_T - r_E)$ over each six-month period. Equation (2) shows that the marginal value of tax-exempt borrowing equals the difference between interest earned at r_T and interest earned at r_E, discounted to the present at the taxable rate of interest.

There is another interpretation of equation (2) with respect to the choice of an appropriate discount factor for municipalities. Equation (2) and the previous discussion indicate that tax-exempt borrowing is a positive net present value project. But, using the tax-exempt rate as a discount factor implies that the borrowing is fairly priced. In other words, the net present value of selling exempt rate is identically equal to zero. Note that this reasoning does not imply infinite arbitrage opportunities: municipalities borrow only to finance a limited number of profitable capital projects. The situation of a municipality is analogous to that of the firm with government-subsidized borrowing. Net present-value gains from borrowing are realized only on some limited quantity of funds.

Equation (3) presents the capital budgeting rule for the municipality: invest capital until the net present value of an additional \$1, financed by borrowing, equals zero. Note that present value is, once again, computed using the taxable rate. Cash inflows from projects and cash outflows to debt holders should both be discounted at the taxable rate of interest.

Before concluding this section, we should discuss another approach to the discount rate question. Gordon and Metcalf (1992) argue that, for spending financed by taxes, the appropriate discount rate is the after-tax taxable rate of the community in question, that is, $r_T(1 - \tau)$, where τ is the "representative" (federal) tax rate of citizens in that particular municipality. The case for this choice is that financing an expenditure from a one-time tax requires citizens to sacrifice an annual flow of $r_T(1 - \tau)$ times the amount of the expenditure.

Even if we put aside any objections with respect to the device of a representative citizen, this line of reasoning implicitly assumes that a municipality can shift tax revenues across time in recognition of the citizenry's opportunity cost of funds. This being the case, however, tax revenues should be advanced as much as possible, since the municipality earns r_T on tax balances whereas individuals earn only the maximum of $r_T(1 - \tau)$ and r_E. But if taxes are advanced so that the municipality invests all tax balances, r_T is clearly the correct discount rate! On the other hand, if the original assumption is not correct, and a municipality cannot advance tax revenues, then the problem reverts to the one developed in this section: minimize the level of taxes that meets spending requirements. And, as has been shown, the discount rate that emerges from this problem is r_T. In short, while Gordon and Metcalf (1992) recommend using the representative citizen's after-tax rate, this article argues that a municipality, taking advantage of the opportunities presented to it, should use relevant market rates for discounting.

APPLICATIONS: REFUNDING AND CHOOSING AMONG CAPITAL PROJECTS

This section presents two important applications of the finding that municipalities should discount future cash flows at the taxable rate of interest. In both cases it is shown that using the tax-exempt rate leads to incorrect or suboptimal decisions.

Refunding[6]

Assume that a municipality has issued a bond with no embedded options, with a \$1 face value, and with an annual coupon of c. Assume that N years remain until maturity and that interest rates have fallen since the time of issue so that $r_E < c$. The market price of the bond, P, determined by tax-exempt yields, exceeds \$1 and is given by

$$P = \sum_{n=1}^{N} \frac{c}{(1 + r_E)^n} + \frac{(1 + c)}{(1 + r_E)^N}$$

$$= \frac{c}{r_E} + \frac{1 - \dfrac{c}{r_E}}{(1 + r_E)^N}$$

The following opportunity is available to the municipality. It may sell P face value of a new, par bond with a coupon r_E to finance the purchase of the outstanding bond for P. This results in annual interest savings of $c - Pr_E = (c - r_E)(1 + r_E)^{-N} > 0$, which may be used to increase the amount of unrestricted funds invested at the taxable rate, r_T. However, N years from now, the municipality must pay off the face value of the new bond issue, which exceeds the face value of the outstanding issue by $P - 1 = (c - r_E)/r_E + (1 - c/r_E)(1 + r_E)^{-N}$.

6. This subsection assumes that municipalities refund their debt by selling new debt to finance a tender offer for the outstanding debt. In fact, municipalities defease, or prerefund, their debt by selling new debt to finance the purchase of U.S. Treasury securities that, in escrow, guarantee payment on the outstanding debt. While these two refunding techniques are different, the economic intuition behind their desirability is the same.

It is easy to demonstrate that this set of transactions proves profitable for the municipality as long as $r_T > r_E$. The intuition behind this result is that refunding enables the municipality to shift outlays from coupon dates to the maturity date at the rate implicit in the municipal bond market, r_E, while the funds made available on coupon dates can be invested to maturity at the higher rate r_T.

Using a discount rate of ρ, the present value of the refunding operation is the present value of the interest savings minus the present value of the shortfall at maturity. Mathematically, this present value is given by

$$\frac{(c - r_E)}{(1 + r_E)^N} \sum_{n=1}^{N} \frac{1}{(1 + \rho)^n} - \frac{\dfrac{(c - r_E)}{r_E} + \dfrac{\left(1 - \dfrac{c}{r_E}\right)}{(1 + r_E)^N}}{(1 + \rho)^N} \tag{5}$$

$$= \frac{(c - r_E)}{(1 + r_E)^N (1 + \rho)^N} \left[\frac{(1 + \rho)^N - 1}{\rho} - \frac{(1 + r_E)^N - 1}{r_E} \right]$$

As shown in the previous section, the correct choice for ρ is r_T. And, as expected, the term in brackets is positive for r_T, as it is for all $\rho > r_E$. Note, however, that if cash flows were discounted at the rate r_E, the term in brackets would equal zero and the refunding operation would not appear to be profitable.

Choosing Among Capital Projects

Assume that the municipality has a choice between two mutually exclusive projects that require equal outlays today but that produce different cash flows over time. Project A produces cash flows $\{d_{An}\}$ while project B produces cash flows $\{d_{Bn}\}$, $n = 1,2, \ldots N$. Revenues from projects are not considered funds raised through tax-exempt borrowing, and, therefore, they may be reinvested at the taxable rate.

Project A should be strictly preferred to project B if and only if the accumulated value N years from now is larger for project A—that is, if and only if

$$\sum_{n=1}^{N} d_{An}(1 + r_T)^{N-n} > \sum_{n=1}^{N} d_{Bn}(1 + r_T)^{N-n}$$

or, when both sides are divided by $(1 + r_T)^N$, if and only if

$$\sum_{n=1}^{N} \frac{d_{An}}{(1 + r_T)^n} > \sum_{n=1}^{N} \frac{d_{Bn}}{(1 + r_T)^n}$$

In words, the municipality should compare the net present value of the cash flows of the two projects using the discount rate r_T. It follows immediately that using a discount rate r_E will, for some sets of cash flows, lead to the wrong capital budgeting decision.

CONCLUSION

The appropriate discount rate for an economic agent is derived from the opportunity cost of a marginal dollar to be received on future dates. Because municipalities can borrow and must sometimes invest at the tax-exempt rate, it has been erroneously concluded that the appropriate discount rate for their future cash flows is the tax-exempt rate. In fact, because municipalities can and do invest funds at the taxable rate, their marginal opportunity cost is the taxable rate. Use of the tax-exempt rate instead can lead to incorrect decisions with respect to refinancing existing debt and choosing among capital projects.

While the analysis of this article has focused on municipalities, it is applicable to any agent with access to subsidized borrowing. Common examples include firms with access to government-subsidized loans, firms with access to the relatively small market for tax-exempt corporate debt, homeowners with access to the "teaser" rates of adjustable rate mortgages, and the more prosaic car buyer with access to a favorable financing arrangement.

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