Creating a Live Yield Curve in the Illiquid Muni Market

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Tax-exempt municipal bonds (commonly known as "munis") are unique to the United States. The size of the muni market is substantial—roughly $3.8 trillion. There are approximately 60,000 issuers and 2 million individual securities (Cusips). In spite of its size, the muni market is illiquid, and consequently does not have a generally accepted live yield curve.

Historically, long-term municipal bonds have been issued as callable any time after 10 years. The initial call price used to be around 102, and it would decline to par within a couple of years. In recent years it has become the norm to set the initial call price to 100.

In the past, munis were priced close to par in the primary market, as is customary in other bond markets. However, during the past decade it has become standard practice to set the coupon of institutional deals to 5%, virtually independently of maturity and credit. The rationale for this unusual practice is discussed below. Because 5% is well above prevailing market yields, the prices of 5% bonds are substantially above par; depending on maturity and credit, they may exceed 125. To summarize, the typical muni bond carries a 5% coupon, and it is callable at par at any time after 10 years (NC-10, i.e., not callable for 10 years).

The standard muni issue has a serial structure, designed to generate a roughly level debt service. Each "maturity" in a series may carry a 5% coupon and be callable, as discussed above. The face amounts of individual maturities are relatively small; thus this practice is partly responsible for the lack of liquidity. Munis may be eligible for advance refunding (Kalotay and Raineri [2016]), whose relevance will be discussed below.

Why 5% NC-10?

The 5% NC-10 structure has appeal to both issuers and institutional investors. The relevant consideration is that 5% bonds are issued at prices well above par (Kalotay [2012]).

Let's begin with the perspective of the issuer. Because a 5% coupon is far above the current market level, 5% bonds are very likely to be refunded—even if rates substantially increase.[1] The resulting savings obviously enhance the stature of the debt manager, in spite of the associated transaction costs, which are naturally welcomed by the numerous intermediaries (underwriters, lawyers, advisors, etc.). A related consideration for bonds eligible for advance refunding is that advance refunding is a free lunch—it can have considerable value, but the issue price of a callable bond is not affected by eligibility for advance refunding (Kalotay and Raineri [2016]).

Turning to the perspective of the institutional investor, the salient point is that the price of a discount muni is further depressed due to the tax payable by the marginal buyer at maturity (Kalotay [2014]). In order not to compromise reported performance, institutional investors favor bonds whose prices are unlikely to fall below par in the event interest rates rise, and 5% bonds achieve this goal. Another factor favoring 5% bonds is that they are likely to be advance refunded too early, with the commensurate transfer of option value from the issuer to the investor (Kalotay [2011]). Tax-savvy investors recognize that bonds purchased near par are unsuitable for tax-loss selling, because if rates rise the market prices can be considerably lower than their "hold" values (Kalotay [2016c, 2016d]).

Institutional Details

The municipal market is regulated by the MSRB. The prices of transactions are reported to MSRB's Electronic Municipal Market Access (EMMA) within 15 minutes and are available to market participants.

The conventional stated yield of a municipal bond is yield to worst (YTW), i.e., the lower of yield to call (YTC) and yield to maturity (YTM). Because most 5% bonds sell at a premium, their YTW is normally the YTC.

YIELD CURVE BASICS

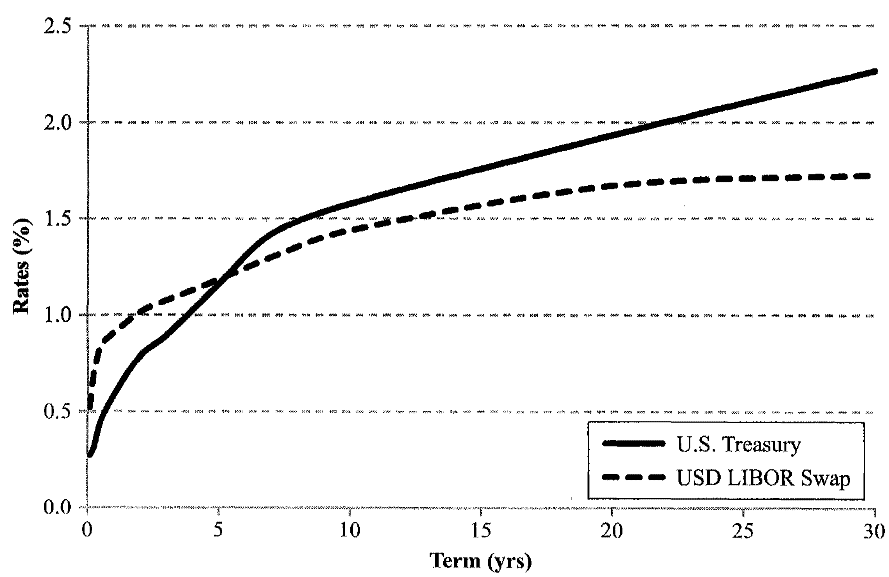
Why YCs Are Needed

Yield curves are essential to the analysis of investment-grade fixed income securities, such as bonds and mortgage-backed securities. Traders and investors quote bond prices as a yield spread to a particular point on a live YC (e.g., YTM is the five-year Treasury rate + 50 bps), or as an OAS (e.g., the OAS relative to the swap curve is 75 bps). The borrowing rates of corporate issuers are specified as spreads to a benchmark (e.g., the two-year spread is 40 bps, the five-year spread is 50 bps, etc.) The resulting borrower-specific yield curve is used for structuring new issues, discounting, etc.

Taxable YCs

In the United States, the commonly used YCs are the Treasury curve and the LIBOR swap curve; their range stretches from short-term (less than 1 year) to 30 years.[2] Both are derived from live market prices. Because the yields of the Treasury curve are based on the prices of actively traded benchmark bonds, which may be above or below 100, the Treasury curve is not a true par YC. On the other hand, the swap rates are at-the-market dealer quotes.

EXHIBIT 1
Benchmark Yield Curves from Taxable Market



Source: Bloomberg, August 25, 2016.

Standard Muni Benchmarks

In light of the industry's preference for 5% NC-10 bonds, it is not surprising that the standard muni benchmark yield curve is specified by the yields of such bonds. The yields for the first 10 years are YTMs, and starting in the 11th year they are YTWs. Several vendors distribute 5% NC-10 curves for AAA-rated issues; some also provide such curves for weaker credits.

Vendors may also provide curves other than 5% NC-10 curves. For example, the Bloomberg benchmark curve assumes that the coupons are 5%, but that the bonds are optionless. It is notable that, given the prevalence of the NC-10 structure, most users are unaware of the distinction. Another variant is a par NC-10 curve, in which case the YTCs and the YTMs are identical.

Currently none of the benchmark YCs are live. They are derived by a combination of "consensus" information (i.e., survey of market participants), analysis of recent trades from EMMA data, and proprietary methodology. Benchmark YCs are usually published daily, toward the end of the trading day.

ANALYTICAL DEFECTS OF CALLABLE BENCHMARK CURVES

In light of the lack of analytical rigor in their construction, the benchmark muni yield curves, not surprisingly, are often deficient. Two common deficiencies will be discussed below: they are not arbitrage-free, and the implied optionless yield curve is unrealistic.

Failure to be Arbitrage-Free

We establish below the arbitrage conditions for the two types of callable benchmarks, one being a par curve, the other a fixed coupon (5%) curve.

Claim 1: In order to be arbitrage-free, the yields (coupons) of a par callable curve, beyond the non-call term, must increase as the maturity increases.

Proof: Denote by C_i the coupon (or yield) for maturity i. Suppose $C_k > C_{k+1}$. Consider the perspective of a borrower who needs funds for k years. If $C_k > C_{k+1}$, the borrower should sell, i.e., issue (k + 1)-year bonds, instead of k-year bonds. The proceeds will be identical, and during the first k years the interest payments will be lower. At the end of the k-th year, when the k-year bond would have matured, the borrower can call the (k + 1)-year bonds at par. Q.E.D.

Claim 2: In order to be arbitrage-free, the prices implied by the yields of a fixed coupon (5%) callable curve, beyond the non-call term, must decrease as the maturity increases (Kalotay and Dorigan [2009]; see also Kalotay [2016a]).

Proof: Denote by P_i the price of the bond maturing at time i. Suppose $P_k < P_{k+1}$. Consider the perspective of borrower who needs funds for k-years. If $P_k < P_{k+1}$, the borrower should sell (k + 1)-year bonds, instead of k-year bonds. The proceeds of the (k + 1)-year bonds will be greater, and during the first k-years the interest payments will be identical. At the end of the k-th year, when the k-year bond would have matured, the borrower can call the (k + 1)-year bonds at par. Q.E.D.

A possible reason why the 5% benchmark curves for lower credit often fail the arbitrage-free test is that their yields are obtained by adding fixed spreads to the risk-free curve, without considering the dollar prices implied by the resulting yields.

Implied Optionless YC Is Unrealistic

Callable benchmark curves are always smooth (see Exhibit 2), and as such they appeal to the eye. But should they be smooth, considering that yields up to Year 10 are YTMs, and beyond Year 10 are YTCs?

A callable curve implies an optionless equivalent par curve (and vice versa). The implied optionless yields are calculated as follows. First, using the so-called bootstrapping method, determine par rates and corresponding discount factors for the first 10 years from the prices of the optionless bonds to Year 10. Next, estimate the 11-year optionless rate, given the price of the 11-year callable bond. This calculation requires that we value the call option, which can be determined using an interest rate process (we use Black-Karasinski), and a realistic volatility parameter. Given these, we iterate the 11-year par optionless rate until it "explains" the price of the 11-year callable bond. Optionless rates beyond 11 years can be estimated in the same fashion.

What volatility is reasonable for this exercise? The swaption volatility for a like lockout and maturity (here, a swaption on a one-year swap exercisable in 10 years, currently about 36%) provides an upper bound. The Commonwealth of Massachusetts (see Burton [2013]) uses a more reasonable 15% volatility for refunding decisions.

Exhibit 2 below displays a typical 5% NC-10 curve, and the implied optionless (NCL, i.e., non-call life) rates, based on the Black-Karasinski process at various volatilities. In every case, the optionless curves have a conspicuous kink near the 10-year point, although the kink becomes less pronounced as the volatility increases. The relevant conclusion is that if the callable curve is smooth, the implied non-callable curve has an unrealistic shape; non-callable curves with a permanent kink strain credulity. We conclude that smooth callable curves are not to be trusted.

The same conclusion can be reached by deriving the callable curve implied by a smooth optionless curve. The implied callable curve will have a kink near the 10-year point (see Exhibit 3).

Misuses of Callable Benchmarks

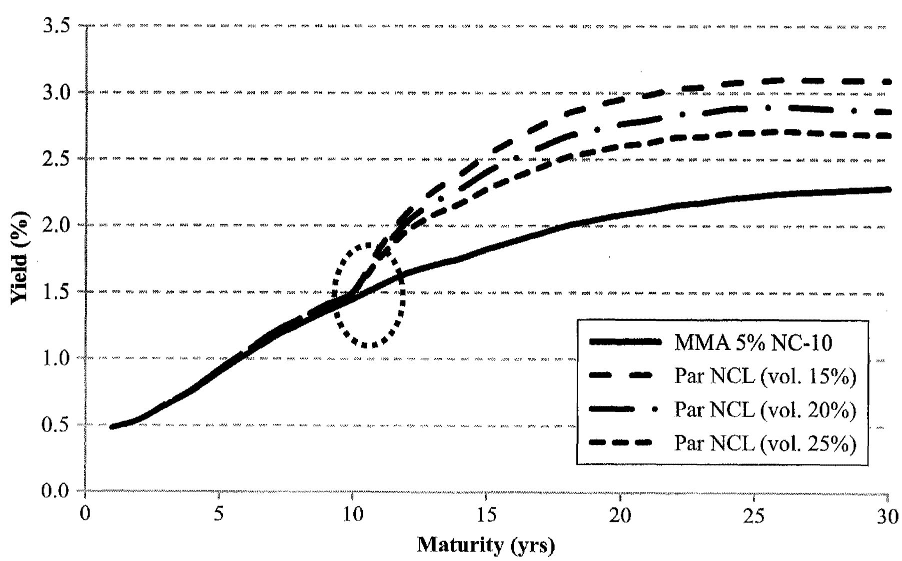
Typical bond analysis, such as estimating duration, forward rates, or relative values, requires an optionless benchmark YC. However, standard muni curves are callable, and converting a callable curve into one that is optionless requires effort. This may explain why many analysts use the callable curve in its raw form. This shortcut can lead to severely misleading results, as will be demonstrated below (Kalotay [2015]).

Suppose that we want to analyze the "benchmark bonds," i.e., the bonds whose yields define a 5% NC-10 curve. Relative to the curves they define, the prices of these bonds should be fair. Specifically, their option adjusted spread (OAS) to the curve should be uniformly 0. Exhibit 4 displays the resulting spreads for the bonds whose yields are shown in Exhibit 2. Instead of being 0, the spreads of the longer-term bonds turn out to be large and positive (Kalotay [2016b]).

The resolution to this apparent paradox is to first convert the NC-10 curve into an NCL curve, as described above, and then use the NCL curve for the analysis. For the purposes of the spread analysis of the benchmark bonds, any reasonable volatility will do: the spreads will be identically zero, indicating that the prices and the optionless curve are in perfect harmony—see the dot-dash line at the bottom of Exhibit 4. Of course, choosing the right volatility is essential for spread analysis of non-benchmark bonds.

There are two critical takeaways about the current state of affairs regarding muni benchmark curves. First, smooth callable curves have an inherent flaw: they imply a kink in optionless rates just past the 10-year point. Second, a callable curve in its raw form should never be used for bond analysis. For sensible results, it must first be converted to an optionless curve, using a reasonable volatility.

EXHIBIT 2
Smooth Callable Curve Implies Kink in Optionless Curve



Source: Municipal Market Advisors, August 25, 2016.

As mentioned earlier, Exhibit 4 displays the spreads (as defined by OAS) of the benchmark 5% bonds against the benchmark curve they define. These spreads on the long end are large and positive, indicating that the benchmark bonds are "cheap." The exhibit also displays the spreads against the implied optionless par curve (options stripped and OAS calculated at 15% vol). As expected, in this case the spreads are identically 0, indicating that the bonds are fairly priced.

THE AP/MBIS BENCHMARK YIELD CURVE

The recently announced initiative by the Associated Press (AP) and the Municipal Bond Information Services

EXHIBIT 3
Smooth Optionless Curve Implies Kink in Callable Curve

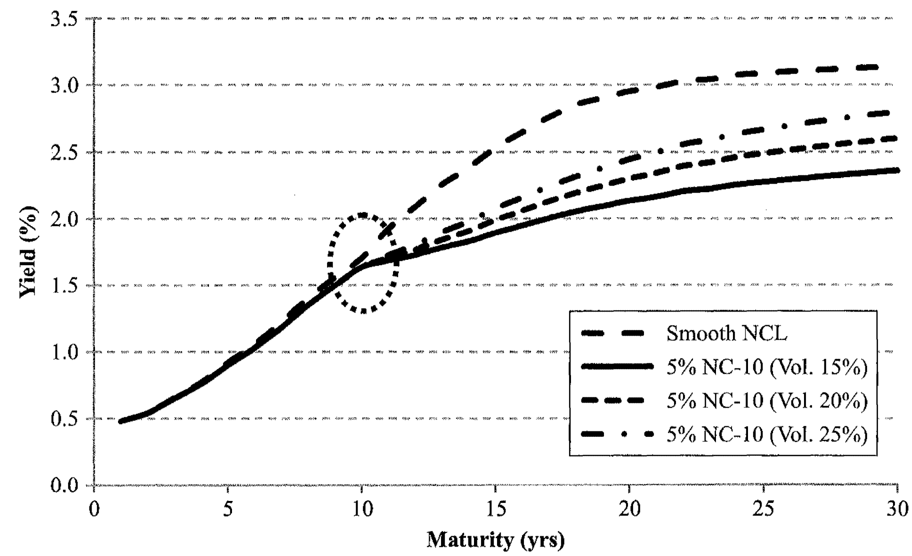
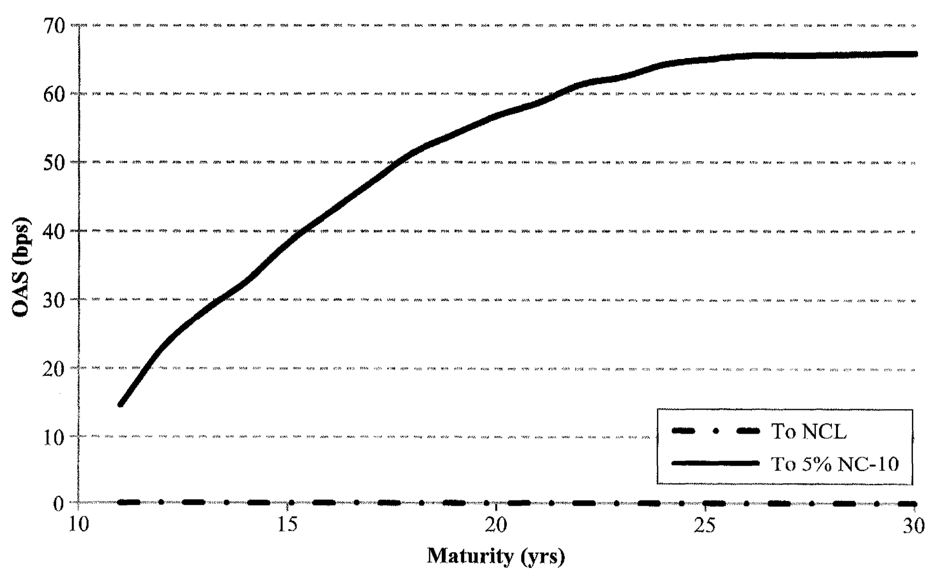


EXHIBIT 4
Direct Use of 5% NC-10 Curve for OAS Gives Absurd Results



(MBIS) is a significant step toward a live market-based muni benchmark curve. According to an April 11, 2016, press release (Associated Press/MBIS [2016]), "The AP/MBIS Bond Index will be the only benchmark based on observable, intraday pre-trade and trade data ..."

The AP/MBIS yield curve is based on dealer quotes (primarily ask prices) for roughly 4,000 actively traded bonds, with credit ratings ranging from AAA to A-. The constituent bonds are updated monthly. Currently the yield curve is refreshed roughly every hour. The AP 10-year yield is already reported by media; a link to the October 7 AP release, as reported by Fox Business, is included in the References (Associated Press [2016]).

In addition to a benchmark yield curve, plans also call for providing credit-specific yield curves (ranging from AAA to BBB) and for an index value based on the prices of a portfolio of actively trading bonds. The latter is intended to provide retail investors with a more understandable indication of market moves than changes in a benchmark yield.

The AP/MBIS curve promises to be an improvement over existing offerings in several ways:

- Constituent selection, bond pricing, and curve construction are fully documented, automated processes that seek to avoid subjectivity and provide reproducibility.
- The process is rigorous and the availability of the underlying pricing and market data enable a thorough attribution of market movements. Plans call for providing attribution based upon key rate durations.
- The curves are constructed using the CurviLinear™ methodology (described below), which allows the proper handling of heterogeneous optionality among the constituent bonds, as well as the construction of an equivalent optionless par curve.
- The curves are produced intra-day, allowing the market to react more effectively to changes in market condition throughout the day.
- The curves are distributed to financial institutions directly, to the financial press, and to the general public (through third party tools by the Associated Press).

THE CURVILINEAR™ SOLUTION TO THE ANALYTICAL CHALLENGE OF A LIVE MUNI CURVE

Overview

The basic task is to build yield curves from bond prices (usually specified as yields, rather than dollar prices). This calculation is accomplished by the CurviLinear™ library, developed by Kalotay Analytics. CurviLinear™ performs a multidimensional nonlinear regression between the vector of yields comprising a yield curve and the vector of known bond prices.

The key component of the regression calculation is an OAS pricing engine (Kalotay, Williams, and Fabozzi [1993]), discussed in some detail below. The pricing engine is initialized using a par optionless yield curve and a specified volatility. Given these inputs, along with the terms of a bond (dated and maturity dates, coupon, call dates and prices, etc.), CurviLinear™ computes the value of the bond at an OAS of 0 bps. The goal of CurviLinear™'s regression analysis is to find the "best" par yield curve, in the sense that the resulting bond values at 0 bps OAS match the input prices as closely as possible. Finally, CurviLinear™ transforms the resulting best optionless par yield curve into a market-convention 5% NC-10 curve.

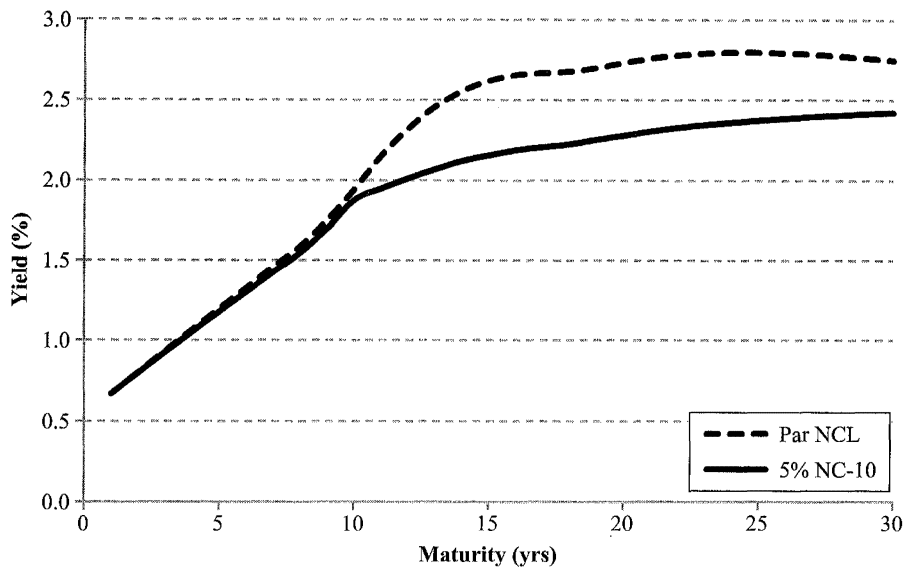
Exhibit 5 displays the par NCL curve and the corresponding 5% NC-10 curve. Note that the par curve is reasonably smooth, and the NC-10 curve has a predictable kink around Year 10.

Although CurviLinear™ can be used for a range of asset classes (corporates, sovereigns, etc.), it is especially useful for municipal bonds, where there is no widely accepted, objective, and systematic algorithm for deriving a market-driven yield curve. As indicated earlier, the most widely used yield curves are built using non-transparent subjective means.

CurviLinear™ provides a systematic and objective way to convert a collection of prices for a heterogeneous set of bonds (different coupons, maturities, and call features) into a "best fit" yield curve. It has the following desirable features:

- Reasonably shaped yield curves with a minimum of wiggles, even if there are gaps in the maturity spectrum of the input bonds.

Exhibit 5
AP/MBIS Index Curve



Source: AP/MBIS, September 16, 2016.

- Different sections of the yield curve (short, intermediate, long) move essentially independently. This is in contrast to spline curves, where a small movement in one section can force a large move in another section, due to the requirement that the curve remain differentiable at the spline knots.

Constructing the OAS Pricing Engine and Pricing a Bond

The OAS pricing engine built by Curvilinear™ is a standard short rate lognormal interest rate model (also known as a Black-Karasinski model) with constant volatility (currently roughly 20%). The OAS pricing model is constructed by converting the specified par yield curve into a vector of discount factors and a corresponding forward rate curve, and then using the forward rates and volatility to construct a multinomial interest rate lattice.

A particular source of complexity associated with tax-exempt bonds is that prices below par are further depressed by unfavorable tax treatment: if held to maturity, the gain resulting from purchase at a discount is taxed as ordinary income. Accordingly, the standard OAS-based valuation needs to be enhanced to accommodate tax effects. This is accomplished by the so-called tax-neutral approach (Kalotay [2014]).

The OAS pricing engine computes the bond value given 0 bps OAS as follows:

- Derive the schedule of after-tax cash flows.
- Compute the 0 bps OAS value of the cash flows by summing the discounted values of the cash flows.
- Compute the value of the call option using the interest rate lattice.
- Dirty price of bond = (discounted value of cash flows) – (value of call option).
- Clean price of bond = (dirty price of bond) – (accrued interest).

Nonlinear Regression Analysis

CurviLinear™ makes better and better guesses at the yield curve such that when used in the OAS pricing engine, the computed bond values match the input prices as closely as possible. This is done by minimizing the sum of squares of the price errors (input price minus computed value).

The highly complex multidimensional nonlinear optimization process used by CurviLinear™ relies on open source C++ optimization software called Ceres Solver, which was developed and is actively maintained by Google software engineers (see http://ceres-solver.org/).

SUMMARY

In spite of its considerable size, the U.S. municipal bond market is illiquid, and historically it has lacked a live benchmark yield curve. Currently available yield curves are largely derived by polling market participants, and they are distributed once or twice daily.

Standard municipal yield curves are based on hypothetical AAA-rated 5% bonds, callable at par after 10 years. They are frequently defective, not being arbitrage-free, and implying unrealistic optionless par curves. Callable benchmark curves are a source of confusion for practitioners, who fail to strip out the optionality and use them as if they were optionless.

The recently announced AP/MBIS yield curve is a sharp departure from the traditional approach. In particular, its methodology is transparent. The AP/MBIS curve is based on dealer-provided ask prices of selected actively traded bonds, and is disseminated several times a day. Preliminary indications are that this new yield curve could become a superior alternative to those currently available.

Construction of a robust yield curve from market prices is analytically challenging. In particular, the prices of discount munis are further depressed by their unfavorable tax treatment. The analytical tool is the CurviLinear™ algorithm, which is driven by a tax-neutral OAS-based valuation engine. Using a non-linear optimizer, CurviLinear™ solves for the optionless par yield curve that minimizes the deviation between the dealer-indicated prices and the fitted bond values. The resulting optionless yield curve is then converted to the standard 5% NC-10 curve, in accordance with current practice.

ENDNOTES

[1]In "Taming Premium Bonds," Smith [2016] argues for premium call prices declining to par to mitigate this effect.

[2]Swap rates are quoted for 40-year and 50-year terms as well.

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