Journal Of Investment Management, Vol. 14, No. 4, (2016), pp. 1–10

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AFTER-TAX PORTFOLIO VALUE: THE MISSING TAX OPTION

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After-tax performance measurement requires a rigorous definition of after-tax portfolio value, which is also a prerequisite for effective portfolio management.

The focus of this paper is the tax option, which is the right to execute tax-beneficial transactions. This option is a critical component of after-tax portfolio value. Some of the proposed definitions incorporate the intrinsic value of the tax option. However, the time value of the option has not been explicitly considered previously. Incorporating the time value of an option is in line with contemporary finance theory, and it provides a more accurate assessment of after-tax value. The tax option can be valued using standard models appropriate to the asset under consideration, but incorporating investor-specific parameters such as tax rates and mortality rates.

1 Introduction

After-tax performance measurement requires a rigorous definition of after-tax portfolio value. An understanding of after-tax value is also a prerequisite for effective management of taxable portfolios. Considerable thought has gone into the pros and cons of alternative definitions of after-tax value; an excellent discussion of historical developments and the merits and deficiencies of proposed alternatives is provided by Horan *et al.* (2008).

As observed by Horan *et al.* (2008), "... Performance based on pre-liquidation value ignores the impact of accrued but unrealized tax liabilities arising from portfolio management, but performance based on post-liquidation values forces the immediate recognition of tax liabilities that can reasonably be delayed." The article proposes a measure of after-tax performance that conforms to neither extreme, by accounting for future (but as yet unrealized) tax liabilities. The article emphasizes that a realistic definition should be based on investor-specific parameters, including applicable tax rates and expected holding period, and that some of these parameters are uncertain—the actual tax rate may differ from the expected rate, and the holding period depends on the

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mortality of the individual investor. The article illustrates the sensitivity of the portfolio's value to investor-specific parameters within a non-stochastic framework. However, it does not take into consideration potential tax-beneficial transactions during the holding period.

The focus of this paper is the tax option, which is the right to execute tax-beneficial transactions. The tax option is a critical component of the after-tax value of a portfolio. While some of the proposed definitions incorporate the intrinsic value of the tax option, the time value of the option has not been explicitly considered previously. Incorporating the time value of an option is in line with contemporary finance theory, and it provides a more accurate assessment of after-tax value. The tax option can be valued using models appropriate to the asset under consideration, but incorporating investor-specific parameters.

These concepts, including the application of the tax option to product design, will be demonstrated for tax-exempt municipal bonds. Such bonds are particularly suitable for this purpose, because they are held virtually exclusively in taxable accounts. It is obvious, however, that incorporating the tax option into the value of the portfolio is applicable to any asset class.

2 Fundamental concepts

Our proposed definition of after-tax portfolio value includes two building blocks, namely 'hold value' and 'tax option'. We begin with a discussion of these concepts. Both are investor-specific; they depend on the assumed tax rates and investment horizon.

Hold value is the present value of the after-tax flows assuming a 'buy-and-hold strategy' to the horizon. Stein (1998) advocates this concept for after-tax value, without actually referring to it as such. In essence, he recommends adjusting the present value of the cashflows by future tax liabilities arising from unrealized gains or losses. Note that the future cashflows may be uncertain, particularly if the security has to be liquidated prior to the horizon.

What is the relationship between hold value and market price? Market price is the present value of the cashflows from the perspective of the marginal investor. Thus in the complete absence of taxes, market price would be a surrogate for hold value. However, as we will see, taxes can complicate matters.

First, assume that the market price is not affected by taxes. In that case, the hold value of the current investor is the market price adjusted by the present value of the taxes payable. An interesting, related consideration is what discount rate should be applied to tax payments. Should the discount rate depend on the risk of the security? In the examples for municipal bonds below, we discount bond flows and tax flows using the same discount rates.

But what if the market price incorporates taxes payable by the marginal investor? Such is the case with tax-exempt municipal bonds selling below par: the gain resulting from buying at a significant (non-*de minimis*) discount is taxed as ordinary income at maturity, and this depresses the market price. The tax effect can be determined by tax-neutral valuation (Kalotay, 2014), which also allows us to determine the unobservable pre-tax value of the security. The hold value can be obtained by adjusting the pre-tax value by the present value of the tax due, which depends on the date and price of the investor's purchase. In summary, the market price and the pre-tax value of the cashflows may differ. Note, however, that the market price is required for determining the pre-tax value.

Let us turn to the tax option, which is defined as the right to execute cashflow-beneficial transactions. Selling a security is cashflow-beneficial if the after-tax proceeds exceed the hold value. We note that *the cashflow benefit is independent of how the proceeds are reinvested*.

The tax option is usually mentioned in the context of 'tax-loss harvesting'. The motivation behind tax-loss harvesting is to recognize a loss for tax purposes. Reinvestment of the after-tax proceeds in a 'like' security increases the pre-tax value of the portfolio, without affecting its risk profile. The transaction is cashflow beneficial provided that the hold value of the new portfolio exceeds that of the original one. A critical consideration is the tax payable on the securities acquired upon reinvestment.

As with any option, the tax option has intrinsic (exercise) value and time value. Current literature considers only the intrinsic value, which is the benefit from immediate sale. By definition, the intrinsic value is non-negative. In summary:

Intrinsic value of the tax option = *max* ((*after-tax proceeds – hold value*), *0*)

We now turn to the central topic of this paper, namely the time value of the tax option. Even if the option has no intrinsic value, under 'favorable' market movement the option could move into the money. Because 'favorable' in this context typically refers to declining market prices, the term needs to be clarified. The point of reference is the value of a like unmanaged portfolio: the steeper the price declines the greater is the potential tax benefit, which increases the difference between the values of the managed and unmanaged portfolios.

The intrinsic value of the tax option is defined unambiguously. Defining the time value can be quite complex, because it depends on the reinvestment strategy, and in turn on the tax option acquired upon reinvestment.

Exercise of the tax option is a trade-off: cashflow savings in exchange for forfeited option value. The challenge is to quantify the forfeited option value. Reinvestment of the proceeds, as assumed below, automatically gives rise to a new tax option, at no cost to the investor. *The value of this new option should also be incorporated into the sale decision*. As stated above, the reinvestment is assumed to be a 'like' security; this ensures that the asset allocation and the risk characteristics of the portfolio remain the same. Thus sale and reinvestment also entails swapping an in-the-money tax option for a like at-the-money option. In fact, we need to contemplate a chain of options, because the newly acquired option may also be exercised. Thus the value of the current tax option should incorporate this 'multigenerational' dimension (Kalotay, 2016a).

Disregarding tax management opportunities beyond the exercise of the option in hand would oversimplify the problem. Because the value of a 'one-time' option is less than that of a 'multigenerational' option, a one-time option would be exercised later, and therefore several potentially favorable opportunities may be forgone during the investment horizon.

How can we determine the value of a tax option, assuming ongoing reinvestments? Keep in mind that a tax option is investor-specific and is not transferable. To begin, note that the value of the tax option comes from the price volatility of the underlying security. Price volatility can be modeled in the customary manner, consistent with currently accepted financial theory. We will illustrate this for municipal bonds, assuming that the interest rates evolve according to the industry-standard Black–Karasinski process.

A stochastic model for the price determines the value of the tax option of a security. The basic rule is to exercise the option when it is worth more dead than alive. The relevant variables are the cashflow savings S, the forfeited tax option value OV_{O} and the new option OV_{N}. The option should be exercised when $S = OV_{\mathrm{O}} - OV_{\mathrm{N}}$ (Kalotay *et al.*, 2007). We can determine the value of a multi-generational tax option by optimally exercising it over the investment horizon. For bonds, the computation can be done recursively (Kalotay and Howard, 2014); for other asset classes, simulation models may be called for.

3 Alternative definitions of after-tax portfolio value

As described by Horan *et al.* (2008), several definitions have been proposed over the years for after-tax portfolio value. We list these definitions below, along with brief observations about each.

1. *Market value* (*also referred to as pre-liquidation value*)

The primary appeal of market value is its simplicity. Because it does not encompass investor-specific considerations, it is obviously not a suitable measure of after-tax value; see Horan *et al.* (2008).

2. *Liquidation value*

In the absence of taxes, liquidation value and market value would be identical. The shortcoming of liquidation value is that sale is indiscriminate, i.e., a sale is assumed whether or not it is cashflow-beneficial. Thus the liquidation value is conservative—it understates the true value of the portfolio.

3. *Hold value*

The drawback of hold value is the opposite of liquidation value: it disregards tax-beneficial sale opportunities. Thus hold value also underestimates true after-tax value.

4. *Intrinsic value* (*defined as hold value plus intrinsic value of the tax option*)

This assumes that a security is liquidated if doing so is beneficial, i.e., if the intrinsic value of the tax option is positive.

Missing from 4. is the time value of the tax option. Even if currently the tax option has no intrinsic value, there may be opportunities to execute tax-beneficial sales in the future. This observation suggests the following definition:

5. *Option-enhanced value* (*hold value plus 'full' value of the tax option*)

This definition is general, and therefore rather vague. As discussed in the previous section, the value of the tax option depends on the reinvestment strategy, specifically on the recognition of the tax option that may be acquired upon reinvestment.

It is informative to compare the above alternatives, excluding the first. The relationship between liquidation value and hold value is uncertain: if liquidation is beneficial then liquidation value exceeds hold value; otherwise hold value is larger. Intrinsic value exceeds liquidation value, because intrinsic value precludes non-beneficial sales. Intrinsic value also exceeds hold value, because hold value does not take beneficial sale opportunities into account. Finally, option-enhanced value is clearly the largest among the alternatives, because it includes both the intrinsic and the time value of the tax option.

4 Illustrations

In the examples below we assume a long (50-year) investment horizon, and that the portfolios consist of single long-term (50-year) tax-exempt optionless (NCL) bonds. As discussed in the

'Implication for Borrowers: Product Design' section below, currently long-term optionless bonds are virtually non-existent. Nevertheless, they provide an insight into the value of long-term tax options, and how option-enhanced portfolio values compare with traditional after-tax measures, namely hold value and intrinsic value.

An intricate aspect of tax-exempt bonds arises from the tax treatment of discount purchases. The resulting gain is taxed at maturity as ordinary income, depressing current market price. This effect is most significant for shorter maturities, although it can be observed even in the case of 50-year bonds.

Table 1 Simulation assumptions.

Interest rate process	Black–Karasinski[1]
Initial optionless yield curve	1-yr 2.0%, 5-yr 2.4%, 10-yr 3.0%, 20-yr 3.25%, 30-yr+ 3.50%
Short-rate volatility	15%
Mean reversion	2
Tax rate assumptions	Income 40%
	Short-term gain/loss 40%
	Long-term gain/loss 20%
Transaction cost	0.25% of par

[1]If the mean reversion is 0, the Black–Karasinski process reduces to the industry-standard log-normal process. A positive mean reversion, here 2, reduces the volatility of the longer-term rates, resulting in more conservative option values.

One of our objectives is to show how the value of the tax option can be determined by combining contemporary asset valuation models with investor-specific assumptions. In the examples below interest rates are assumed to evolve according to the Black–Karasinski process. The resulting yield curves determine the market prices of the bonds, based on tax-neutral valuation using reasonable assumptions about the tax rate of the marginal investor. Note that although the bonds are optionless, the value of the tax option depends on the assumed volatility—here 15%. In general, the higher the volatility, the greater the value of a tax option. Table 1 lists the assumptions.

Figure 1 displays the prices (left axis) and the option values (right axis) of 50-year bonds with coupons ranging from 2% to 6%. The prices, obtained by tax-neutral valuation (Kalotay, 2014), assume that the bonds were issued at par. The prices of bonds with coupons above 3.50% are above par, and they increase linearly with the coupon. The prices dip as the coupon drops below 3.50%, and dip again when the price drops below 87.50, which is the so-called de minimis level for 50-year bonds (below which the discount is taxed as ordinary income). The tax option increases with the coupon/price, because there are increasingly better opportunities to take losses.

Figure 2, which is derived from Figure 1, shows the option value normalized by the purchase price. We observe that the normalized option value continues to increase with coupon (or purchase price), although the effect is less pronounced. The reason is that the prices of bonds trading below par are negatively tax-affected, and this makes them less attractive sale candidates. For bonds selling at a deep discount the normalized option value is roughly 5%, for those selling near par it is in the neighborhood of 6.5%, and for bonds selling at a high premium it rises to nearly 8%. For shorter maturities and/or shorter horizons the option values would be smaller.

Figure 3 displays how instantaneous yield curve shifts affect the hold value, the intrinsic value, and the option-enhanced value of a 3.50% bond purchased at par. Because the bond is purchased at par, both its hold and its intrinsic value are 100% at the unshifted point. This is a degenerate case; as discussed above, the intrinsic value usually exceeds the hold value. The option value in this case is 6.30%, therefore the option-enhanced value is 106.30%. Lower rates increase and higher

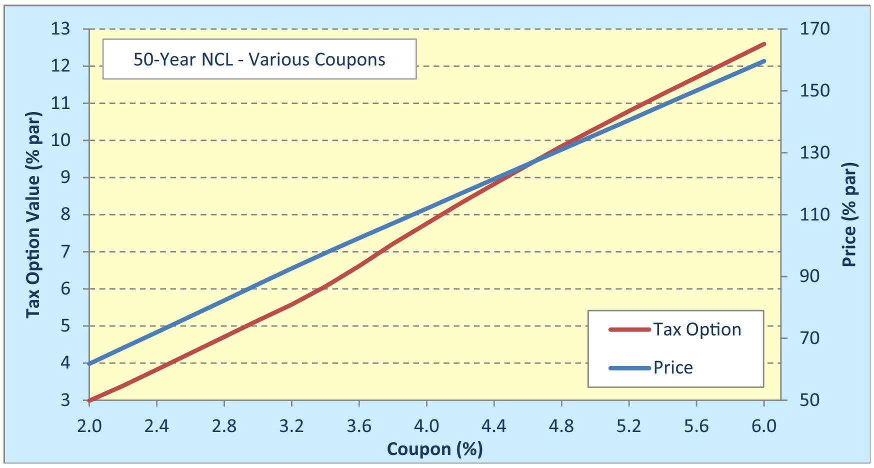

Figure 1 Tax option value increases with purchase price.

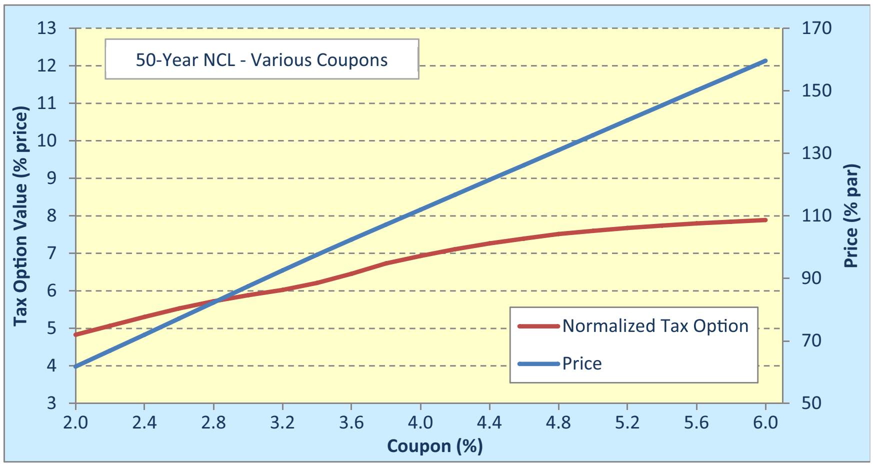

Figure 2 Price and normalized tax option value.

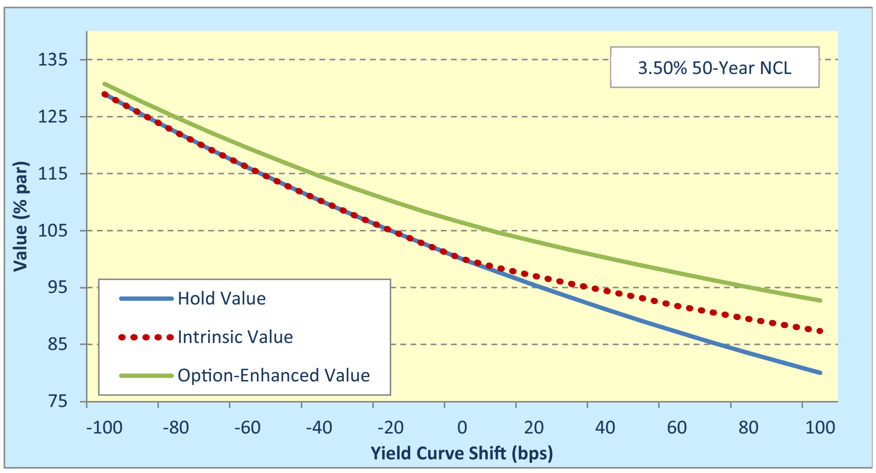

Figure 3 Value given instantaneous shocks to yield curve.

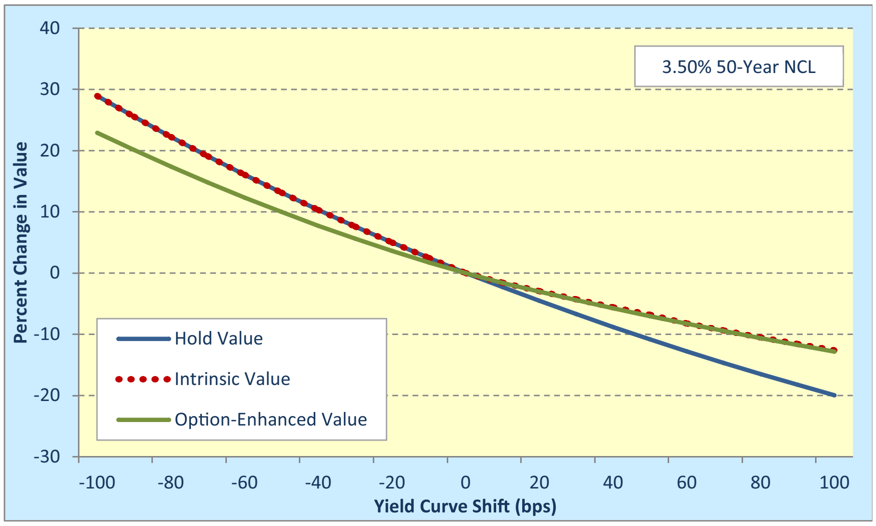

Figure 4 Option-enhanced value moderates performance.

rates decrease the value. If rates decline, the intrinsic value and the hold value are identical. The difference between these and the option-enhanced value narrows, because the value of the option declines. On the other hand, if rates increase and prices decline, the intrinsic value and the option-enhanced value move virtually in tandem, while the hold value underperforms relative to both of them.

Figure 4 displays the 'instantaneous performance', i.e., the effect of yield curve shifts as a percentage change relative to the original value. Incorporating option value clearly moderates performance. For example, if the yield curve declines 100 bps, both the hold value and the intrinsic value increase 29%, while the option-enhanced value increases only 23%. If rates increase 100 bps the hold value declines 20%, while the option-enhanced value declines only 13%, about the same as the intrinsic value.

6. *Implication for borrowers: Product design*

The goal of the manager is to maximize the option-enhanced value of the portfolio. As demonstrated in Kalotay (2016b), certain bonds are more suitable to achieve superior after-tax performance than others. In particular, intermediate-term bonds purchased close to par are unattractive tax management candidates. The reason, as discussed earlier, is that when interest rates rise, the prices of discount bonds are further depressed by the tax on the gain payable by the marginal investors. Such adverse tax treatment does not affect hold values of bonds purchased above par. Therefore, holding a discount bond is preferable to selling it, in spite of the tax savings. Intermediate-term bonds purchased near par can be sold in a tax-beneficial way only if the resulting loss is short-term, i.e., if the sale is within a year of purchase and there is an offsetting short-term gain. The adverse tax treatment is less critical for long-term bonds, because the present value of the tax payable at maturity is smaller.

The above insight has a clear implication for borrowers who wish to appeal to tax-aware investors: they should issue bonds with above-market coupons, making them unlikely to drop below par in the secondary market. Landoni (2016) explores the problem of optimizing the coupon rate in order to maximize the potential tax benefit for investors. According to Landoni, "An optimally issued bond can be worth up to 3.5% more than a par bond, a gain created at the expense of the U.S. Treasury."

Even though long-term munis are almost always callable, Landoni considers only non-callable bonds, and he asserts that from a tax perspective the ideal long-term structure would be a non-callable original issue discount bond. As discussed in Kalotay (2012), municipal market participants, both investors and issuers, have a preference for callable bonds issued with an above-market coupon, currently 5%. The primary impetus for this practice is that the call option usually allows the borrower to advance refund prior to the call date, to the benefit of both the borrower and investors. The market does not extract an extra charge for the advance refunding option, which can be worth several points to the issuer (Kalotay and Ranieri, 2016).

Municipal bonds issued above par are usually callable at 100 at any time after Year 10. Original issue discounts, which will not be considered, are callable at accreting prices. Below, we explore the dependence of the value of the tax option of 15NC10 bonds on the coupon.

The duration of these bonds declines as the coupon increases. Duration is a measure of price volatility, which in turn fuels the value of the tax option. This consideration favors the lower-coupon bonds. On the other hand, lower coupon bonds are more likely to fall to a discount, with the familiar adverse effect of taxes. The relatively short 15-year maturity magnifies the impact of the tax treatment. The other important consideration is callability. The likelihood of call increases as the coupon increases, and this reduces the effective maturity. A sufficiently high coupon guarantees that the bond will be called at the end of Year 10. Finally, we note that the price of the bond increases with the coupon, and in order to compare tax options on an apples-to-apples basis, they should be normalized by the purchase price of the bond.

Figure 5 displays how the normalized value of the tax option depends on the coupon, given the market parameters shown in Table 1. In particular, the applicable short-term capital gains rate is assumed to be 40%. The value initially increases with the coupon, and it peaks slightly over 2%, when the coupon is roughly 5% and the

Figure 5 Value of tax option of 15NC-10 bond peaks at coupon of around 5%.

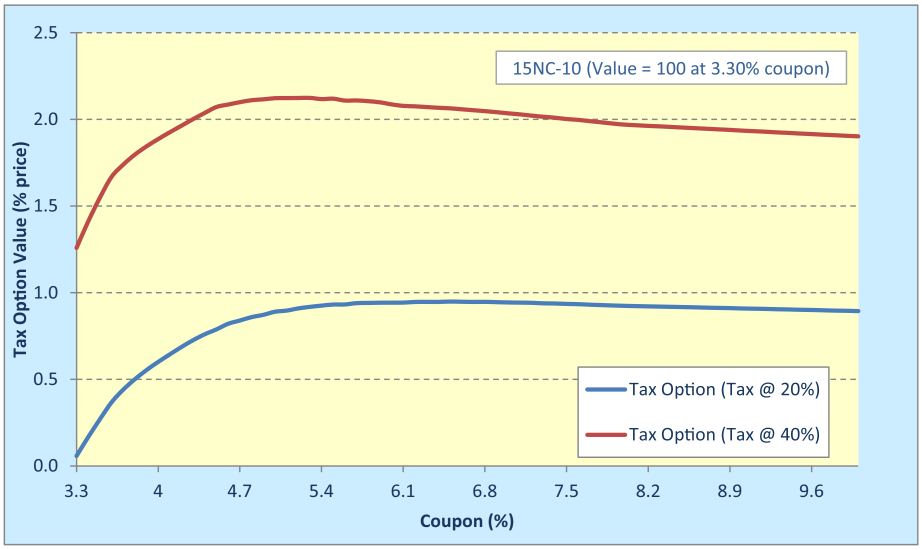

Figure 6 Option value of bonds near par is insignificant without offsetting short-term gains.

corresponding dollar price is 120; beyond 5% the value gradually declines. In short, bonds selling at a reasonable premium are optimal for tax management purposes, while those selling either close to par or at a very high premium are undesirable. The theoretically optimal coupon is remarkably close to the 5% favored by current market issuance practice.

As the maturity is extended the optimal coupon gradually declines. This is partly because the duration (price volatility) increases as the coupon declines, and partly because the adverse impact of the tax on discounts is mitigated by the long effective maturity.

Figure 6 displays the sensitivity of the value of the normalized tax option to the applicable short-term capital gains rate. It shows the values assuming that the rate is 40%, as in Figure 5, and also if it is only 20%. The latter, conservative assumption, is appropriate if short-term losses are netted against long-term gains. As can be seen in Figure 6, the lower tax rate significantly reduces the value of the option. At a 20% rate the value reaches its maximum, slightly below 1%, when the coupon is just above 5%. The tax rate has a striking effect on bonds sold close to par: if the applicable tax rate is only 20%, the tax option is almost worthless.

5 Conclusion

At present there are several alternative definitions of after-tax portfolio value in use. These include the unmanaged ‘hold’ value and ‘intrinsic’ value. Hold value fails to consider tax-beneficial transactions. Intrinsic value partially corrects for this shortcoming, as it recognizes existing tax-beneficial transactions, i.e., it takes into account the intrinsic value of the tax option. However, intrinsic value still disregards potential tax-beneficial transactions, i.e., the time value of the tax option.

We propose a new definition of after-tax portfolio value, namely option-enhanced value. Option-enhanced value includes both the intrinsic value and the time value of the tax option, and it is clearly an improvement over both hold value and intrinsic value.

Quantifying the value of the tax option can be challenging because of several distinct factors. It is investor-specific—it depends on the relevant tax rates and investment horizon. It also depends on the contemplated management strategy; ongoing reinvesting provides additional tax optionality at no additional cost. The price volatility of the security is the primary driver of the value of the tax option. For investor-specific parameters, the value of the tax option can be determined using industry-standard pricing models. The recognition of the tax option can be used by borrowers to design products which appeal to tax-savvy investors. We have demonstrated the procedure for tax-exempt municipal bonds, which, from an analytical perspective, are particularly complex.

The value of a conventional embedded option, such as a call or a put in a bond, is encompassed in the price of the security, and therefore it is automatically reflected in the reported market value of the portfolio. Such is obviously not the case with the tax option. Nevertheless, in order to conform to contemporary finance theory, the value of the embedded tax option should be recognized. Option-enhanced value provides the portfolio manager with a more realistic assessment of a taxable portfolio. While performance measurement based on option-enhanced value may not be practical for external reporting, it is useful for internal purposes, because it incorporates future tax-beneficial opportunities. Furthermore, it has the effect of moderating return relative to those based on alternative definitions of after-tax value.

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Keywords: After-tax value, tax option, portfolio value, after-tax performance, tax-loss harvesting